

Solution #1

$$(a) \quad P(A) = P(B) = P(C) = 1/2$$

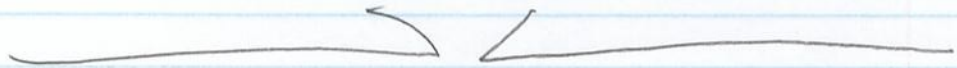
$$A \cap B = \{2, 3\} \quad A \cap C = \{3, 4\} \quad B \cap C = \{3, 6\}$$

$$\Rightarrow P(A \cap B) = P(A)P(B), \quad P(A \cap C) = P(A)P(C), \quad P(B \cap C) = P(B)P(C)$$
$$= 1/4 \qquad \qquad \qquad = 1/4 \qquad \qquad \qquad = 1/4$$

$$\text{Finally } A \cap B \cap C = \{3\} \text{ and } P(A \cap B \cap C) = P(A)P(B)P(C)$$
$$= 1/8$$

A, B, C are statistically independent.

(b) E and F are statistically independent by theorem 3 at page 10 in the notes.



Solution #2

$$(i) P(n_0) = 0.5 \times 0.8 + 0.25 \times 0.2 = 0.45$$

$$P(n_1) = 0.5 \times 0.2 + 0.8/4 = 0.3$$

$$P(n_2) = \frac{0.2}{4} + \frac{0.8}{4} = 0.25$$

somme = 1 check!

$$(ii) P(m_0 | n_1) = \frac{P(m_0, n_1)}{P(n_1)} = \frac{0.5 \times 0.2}{0.3} = 1/3$$

$$P(m_1 | n_1) = \frac{P(m_1, n_1)}{P(n_1)} = \frac{0.2}{0.3} = 2/3$$

$$P(m_2 | n_1) = \frac{P(m_2, n_1)}{P(n_1)} = \frac{0}{0.3} = 0$$

somme = 1 check!



Solution #3

$$\sigma^2 = 9$$

$$\frac{1}{3\sqrt{2\pi \times 9}}$$

$$\frac{1}{3\sqrt{2\pi}}$$

$$\frac{1}{\sqrt{2\pi \times 9}} e^{-\frac{(4-5)^2}{18}}$$

$$m = 5$$

$$\sigma^2 = 9$$

$$(a) \quad Q\left(\frac{7-5}{3}\right) = Q\left(\frac{2}{3}\right) = 0.252492$$

$$(b) \quad Q\left(\frac{3-5}{3}\right) = 1 - Q\left(\frac{2}{3}\right) = 0.7475074$$

$$(c) \quad Q\left(\frac{6-5}{3}\right) - Q\left(\frac{7-5}{3}\right)$$

$$Q\left(\frac{1}{3}\right) - Q\left(\frac{2}{3}\right) = 0.1169488$$

Solution #4

On calcule d'abord

$$\begin{aligned} p_x(y | Z=0.6) &= \frac{p_{xz}(y, 0.6)}{p_z(0.6)} = \frac{p_{xz}(y, 0.6)}{1 - e^{-0.6}} \\ &= \begin{cases} \frac{e^{y-0.6}}{1 - e^{-0.6}} & ; \text{ si } 0 < y < 1 \text{ et } y < 0.6 \\ 0 & ; \text{ ailleurs} \end{cases} \\ &= \begin{cases} \frac{e^y}{e^{0.6} - 1} & ; 0 < y < 0.6 \\ 0 & ; \text{ ailleurs} \end{cases} \end{aligned}$$

On a alors

$$\begin{aligned} P(X < 0.5 | Z=0.6) &= \int_{-\infty}^{0.5} p_x(y | Z=0.6) dy \\ &= \int_0^{0.5} \frac{e^y}{e^{0.6} - 1} dy = \frac{e^{0.5} - 1}{e^{0.6} - 1} = 0.789085 \end{aligned}$$

$$\begin{aligned} P(X > 0.5 | Z=0.6) &= \int_{0.5}^{\infty} p_x(y | Z=0.6) dy \\ &= \int_{0.5}^{0.6} \frac{e^y}{e^{0.6} - 1} dy = \frac{e^{0.6} - e^{0.5}}{e^{0.6} - 1} = 0.210915 \end{aligned}$$

Il faut donc décider que $X < 0.5$ et il y a 78.9% des chances que cette décision soit bonne. somme $\equiv 1$

