Solution #1

(a) $P(A) = P(B) = P(C) = 1/2$

$A \cap B = \{2, 3\}$ $A \cap C = \{3, 4\}$ $B \cap C = \{3, 6\}$

$\Rightarrow \underbrace{P(A \cap B) = P(A)P(B)}_{= 1/4}$, $\underbrace{P(A \cap C) = P(A)P(C)}_{= 1/4}$, $\underbrace{P(B \cap C) = P(B)P(C)}_{= 1/4}$

Finally $A \cap B \cap C = \{3\}$ and $P(A \cap B \cap C) = P(A)P(B)P(C) = 1/8$

A, B, C are statistically independent.

(b) E and F are statistically independent by theorem 3 at page 10 in the notes.

Solution #2

(i) $P(r_0) = 0.5 \times 0.8 + 0.25 \times 0.2 = 0.45$

$P(r_1) = 0.5 \times 0.2 + 0.8/4 = 0.3$

$P(r_2) = \frac{0.2}{4} + \frac{0.8}{4} = 0.25$

somme $\equiv 1$ check!

(ii) $P(m_0 | r_1) = \frac{P(m_0, r_1)}{P(r_1)} = \frac{0.5 \times 0.2}{0.3} = 1/3$

$P(m_1 | r_1) = \frac{P(m_1, r_1)}{P(r_1)} = \frac{0.2}{0.3} = 2/3$

$P(m_2 | r_1) = \frac{P(m_2, r_1)}{P(r_1)} = \frac{0}{0.3} = 0$

somme $\equiv 1$ check!

Solution #3

$\sigma^2 = 9$

$$\frac{1}{3\sqrt{2\pi \times 9}}$$

$$\frac{1}{3\sqrt{2\pi}} \qquad \frac{1}{\sqrt{2\pi \times 9}} e^{-(x-5)^2/18}$$

$m = 5$

$\sigma^2 = 9$

(a) $Q\left(\frac{7-5}{3}\right) = Q(2/3) = 0.252492$

(b) $Q\left(\frac{3-5}{3}\right) = 1 - Q\left(\frac{2}{3}\right) = 0.7475074$

(c) $Q\left(\frac{6-5}{3}\right) - Q\left(\frac{7-5}{3}\right)$

$Q(1/3) - Q(2/3) = 0.1169488$

Solution #4

On calcule d'abord

$$p_X(y \mid Z=0.6) = \frac{p_{XZ}(y,0.6)}{p_Z(0.6)} = \frac{p_{XZ}(y,0.6)}{1-e^{-0.6}}$$

$$= \begin{cases} \dfrac{e^{y-0.6}}{1-e^{-0.6}} & ;\ \underbrace{\text{si } 0<y<1 \text{ et } y<0.6}_{0<y<0.6} \\ 0 & ;\ \text{ailleurs} \end{cases}$$

$$= \begin{cases} \dfrac{e^{y}}{e^{0.6}-1} & ;\ 0<y<0.6 \\ 0 & ;\ \text{ailleurs} \end{cases}$$

On a alors

$$P(X<0.5 \mid Z=0.6) = \int_{-\infty}^{0.5} p_X(y \mid Z=0.6)\,dy$$

$$= \int_0^{0.5} \frac{e^{y}}{e^{0.6}-1}\,dy = \frac{e^{0.5}-1}{e^{0.6}-1} = 0.789085$$

$$P(X>0.5 \mid Z=0.6) = \int_{0.5}^{\infty} p_X(y \mid Z=0.6)\,dy$$

$$= \int_{0.5}^{0.6} \frac{e^{y}}{e^{0.6}-1}\,dy = \frac{e^{0.6}-e^{0.5}}{e^{0.6}-1} = 0.210915$$

somme $\equiv 1$

Il faut donc décider que $X<0.5$ et il y a 78.9% des chances que cette décision soit bonne.