Solution #1

In order to obtain a normalized 3rd order Butterworth low-pass filter we choose R, C, L_1, L_2 in such a way that

$$\left.\begin{aligned} \frac{L_1 L_2 C}{R} &= 1 \\ L_2 C &= 2 \\ \frac{L_1 + L_2}{R} &= 2 \end{aligned}\right\} \quad L_1 = \frac{R}{2} \rightarrow L_2 = \frac{3R}{2} \rightarrow C = \frac{4}{3R}$$

With a $100\,\Omega$ resistor R we obtain:

$L_1 = 50\text{H}$
$L_2 = 150\text{H}$
$C = 13.33\text{mF}$

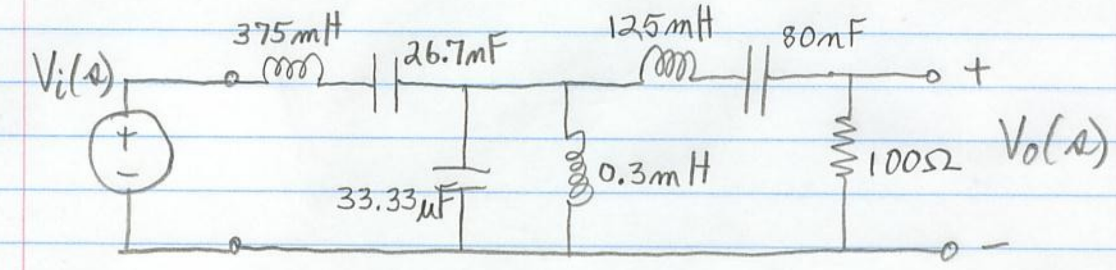
Finally the desired circuit is obtained by applying the "low-pass to band-pass transformation" with $B = 400$ and $\omega_0 = 10000$:

$$L_1 \rightarrow \underbrace{\frac{50}{400}\text{ H}}_{125\text{mH}} \text{ in series with } \underbrace{\frac{400}{(10000)^2\, 50}\text{ F}}_{80\text{nF}}$$

$$L_2 \rightarrow \underbrace{\frac{150}{400}\text{ H}}_{375\text{mH}} \text{ in series with } \underbrace{\frac{400}{(10000)^2\, 150}\text{ F}}_{26.7\text{nF}}$$

$$C \rightarrow \underbrace{\frac{13.33\text{m F}}{400}}_{33.33\mu\text{F}} \text{ in parallel with } \underbrace{\frac{400}{(10000)^2\, 13.33\text{m}}\text{ H}}_{0.3\text{mH}}$$

The circuit is presented on the following page:



The circuit is solved and its magnitude of frequency response is sketched on the following MAPLE session (see next pages).

Solution #2

(a) $P(A) = P(B) = P(C) = 1/2$

$A \cap B = \{2, 3\}$ $A \cap C = \{3, 4\}$ $B \cap C = \{3, 6\}$

$\Rightarrow P(A \cap B) = P(A)P(B)$, $P(A \cap C) = P(A)P(C)$, $P(B \cap C) = P(B)P(C)$
$= 1/4$ $= 1/4$ $= 1/4$

Finally $A \cap B \cap C = \{3\}$ and $P(A \cap B \cap C) = P(A)P(B)P(C)$
$= 1/8$

A, B, C are statistically independent.

(b) E and F are statistically independent by theorem 16 at page 109 in the notes.

Solution #3

$$P(\{-0.5 < X < 1\}) = \int_{-0.5}^{1} p_X(x)\,dx$$

$$= \int_0^1 \frac{1}{2} e^{-x/2}\,dx$$

$$= -\left[e^{-x/2} \right]_0^1$$

$$= -\left(e^{-1/2} - 1\right)$$

$$= 1 - 1/\sqrt{e}$$

$$\approx 0.393469 \approx 39.35\%$$