

Equation différentielle

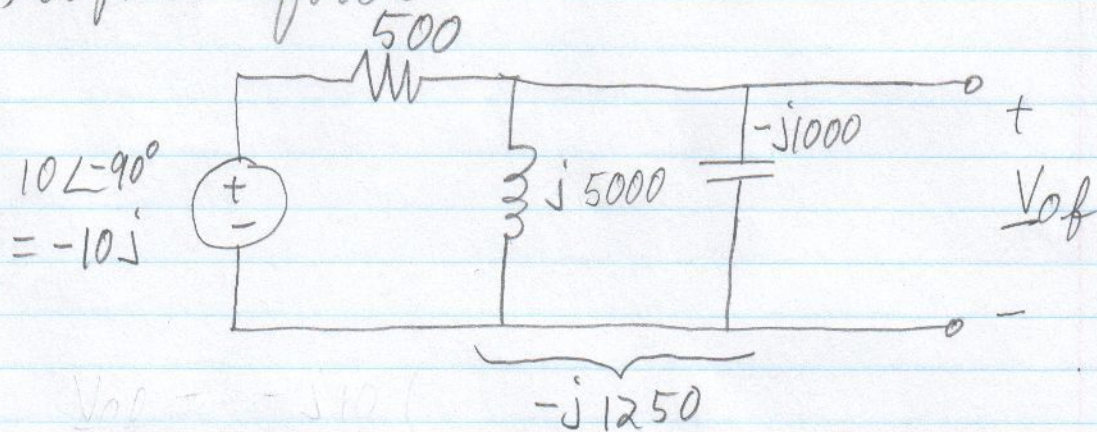
$$\left\{ \begin{aligned} \frac{N_i}{CR} &= \frac{1}{CL} \int v_o dt + \frac{dv_o}{dt} + \frac{v_o}{CR} \\ N_o'' + \frac{1}{RC} N_o' + \frac{1}{LC} N_o &= \frac{N_i'}{RC} \\ N_o'' + 100k N_o' + 500M N_o &= 100k (10 \times 50k \cos(50kt)) \\ &= 50G \cos(50kt) \end{aligned} \right.$$

polyn. caractéristique: $\lambda^2 + 10^5 \lambda + 5 \times 10^8$

$$\lambda_1 = -94721.36$$

$$\lambda_2 = -5278.64$$

(i) réponse forcée



$$\underline{V_{of}} = \frac{-j10 \times (-j1250)}{500 - j1250} = -3.448 - j8.621 = 9.2848 \angle -111.8^\circ$$

$$\Rightarrow v_{of}(t) = 9.2848 \cos(50 \times 10^3 t - 111.8^\circ)$$

(ii) les racines du polynôme caractéristique sont réelles et distinctes.

$$v_{on}(t) = A e^{-94721.36t} + B e^{-5278.64t}$$

$$\Rightarrow v_o(t) = A e^{-94721.36t} + B e^{-5278.64t} + 9.2848 \cos(50 \times 10^3 t - 111.8^\circ)$$

$$(iii) N_0(0) = A + B + 9.2848 \cos(-111.8^\circ) = 0$$

$$N_0'(t) = -94721.36 A e^{-94721.36t} - 5278.64 B e^{-5278.64t} - 9.2848 \times 50 \times 10^3 \sin(50 \times 10^3 t - 111.8^\circ)$$

$$N_0'(0) = -94721.36 A - 5278.64 B - 9.2848 \times 50 \times 10^3 \sin(111.8^\circ) = 0$$

Ce donne 2 équations à 2 inconnues :

$$A + B = 3.44808$$

$$94721.36 A + 5278.64 B = 431040.26$$

La solution est

$$A = 4.615681$$

$$B = -1.167601$$

Finalement

$$N_0(t) = 4.615681 e^{-94721.36t} - 1.167601 e^{-5278.64t} + 9.2848 \cos(50 \times 10^3 t - 111.8^\circ)$$