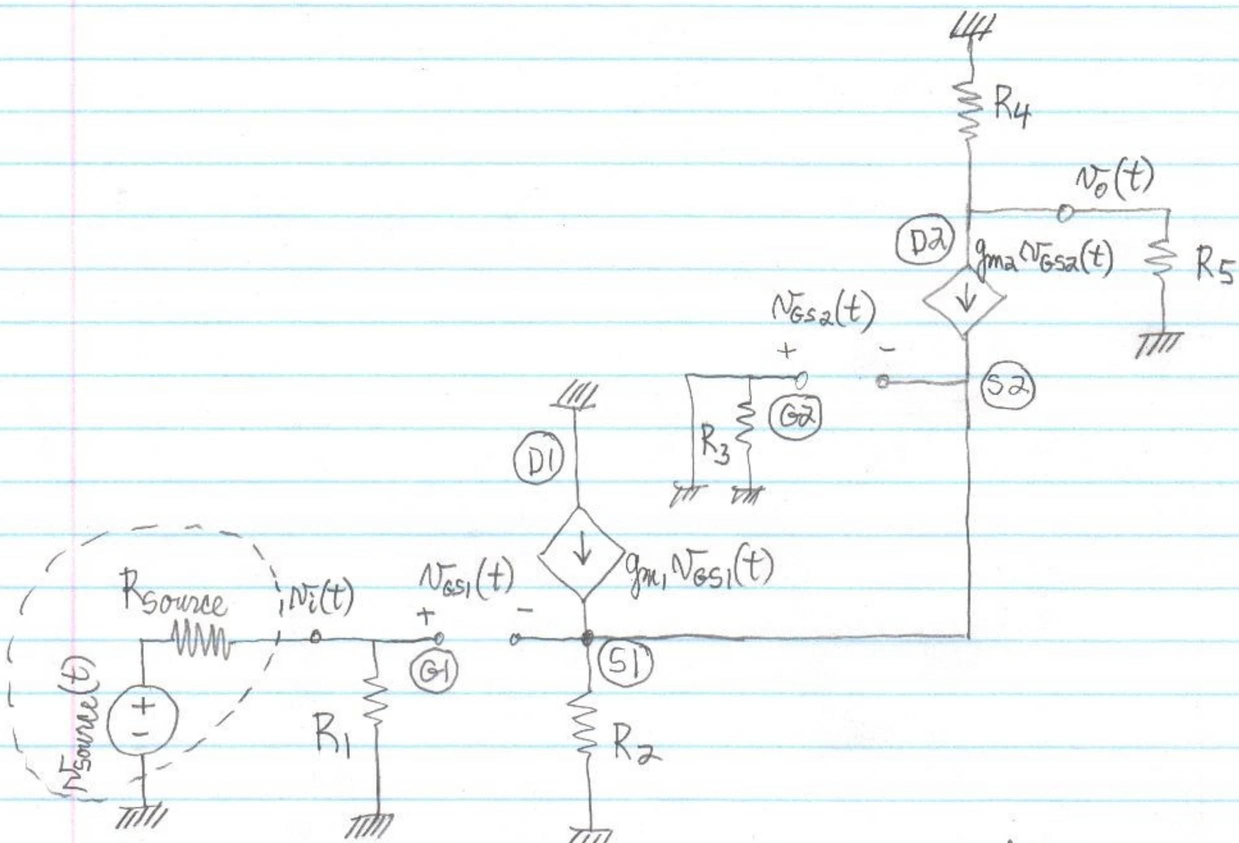
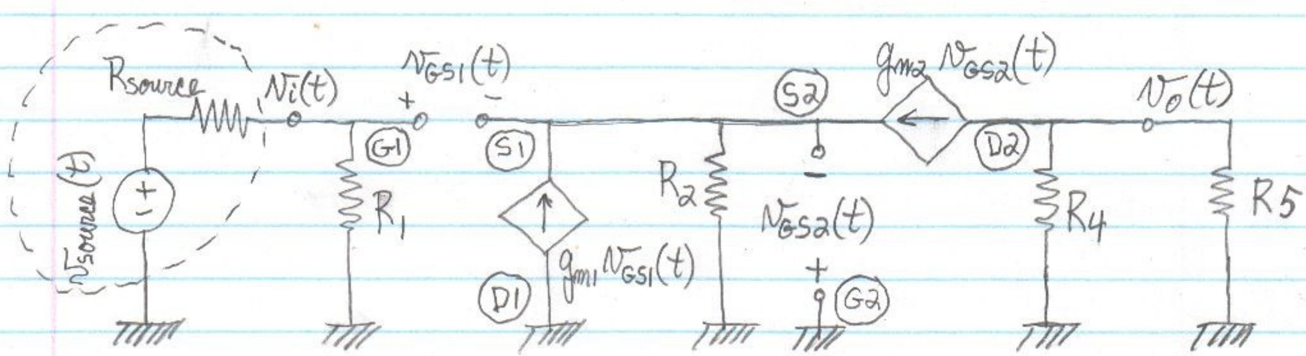
Solution #1

We draw the circuit with the D.C. sources set to 0, the capacitors replaced by short-circuits, and the MOSFETs replaced by their small signal AC model:

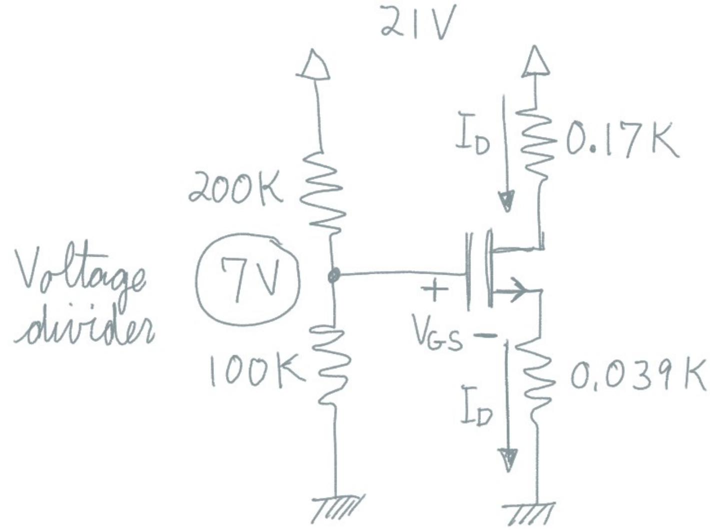


Redrawn in the simplified form, it becomes:



Solution #2

(a) First assume, as suggested that the MOSFET is biased in the active region:



The equations are

$$I_D = 48\,(V_{GS} - 2.1)^2 \qquad (1)$$

$$V_{GS} = 7 - 0.039\, I_D \qquad (2)$$

and by substituting (2) in (1) we obtain

$$\begin{aligned} I_D &= 48\,(4.9 - 0.039\, I_D)^2 \\ &= 48\,(4.9^2 - 2 \times 4.9 \times 0.039\, I_D + 0.039^2\, I_D^2) \\ &= 1152.48 - 18.3456\, I_D + 0.073008\, I_D^2 \end{aligned}$$

The solutions are:

$$I_D = \begin{cases} 90.444 \text{ mA} \Rightarrow V_{GS} = 3.4727 \text{ V} \\ 174.535 \text{ mA} \Rightarrow \underbrace{V_{GS} = 0.19314 \text{ V}}_{\text{eliminate cuz less than } V_t = 2.1 \text{ V}} \end{cases}$$

With $I_D = 90.444$ mA we then obtain

$$V_{DS} = 21 - 0.17 I_D - 0.039 I_D$$
$$= 2.097 \text{ V}$$

which is larger than $V_{GS} - V_t = 1.3727$ V and the MOSFET is consequently biased in the active region as assumed.

(b) With same R_1, R_2 and R_S equations (1) and (2) above still hold and if V_{DS} is to become 10V instead of 2.097V then the MOSFET is clearly active. R_D is then simply obtained from

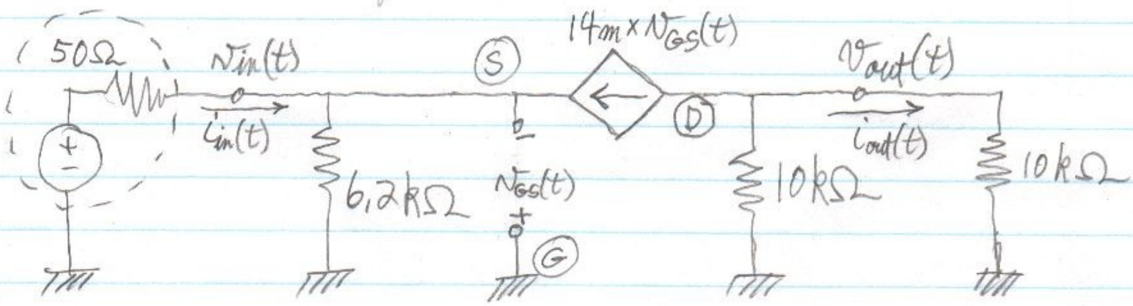
$$V_{DS} = 10 = 21 - (R_D + 0.039) \times 90.444$$

$$\Rightarrow R_D = 82.622\,\Omega$$

Solution #3

From the Q-point we find $g_m = 2K(V_{GS} - V_t)$
$= 2 \times 50 \times (2.64 - 2.5)$
$= 14\,mS$

The AC model of the circuit is



We easily find

(i) $v_{out}(t) = -v_{GS}(t) \times 14\,mS \times (10\,k\Omega \| 10\,k\Omega)$
$\uparrow$ $v_{GS}(t) = -v_{in}(t)$
$= 70\; v_{in}(t) \longrightarrow$ $\boxed{A_V = 70}$

(ii) $i_{in}(t) = \dfrac{v_{in}(t)}{6.2\,k\Omega} - 14\,mS \times v_{GS}(t)$
$\uparrow$ $v_{GS}(t) = -v_i(t)$
$= \dfrac{v_{in}(t)}{6.2\,k\Omega} + \dfrac{v_{in}(t)}{1/14\,m\Omega}$
$= \dfrac{v_{in}(t)}{(6.2\,k\Omega \| (1/14\,mS))} = \dfrac{v_{in}(t)}{70.6\,\Omega} \longrightarrow$ $\boxed{Z_{in} = 70.6\,\Omega}$

(iii) $A_I = \dfrac{A_V Z_{in}}{R_L} = \dfrac{70 \times 70.6\,\Omega}{10\,k\Omega} \longrightarrow$ $\boxed{A_I = 0.494}$

(iv) $A_P = A_V \times A_I \longrightarrow$ $\boxed{A_P = 34.59}$

(V) $Z_{out} = V/I$ in the circuit below: