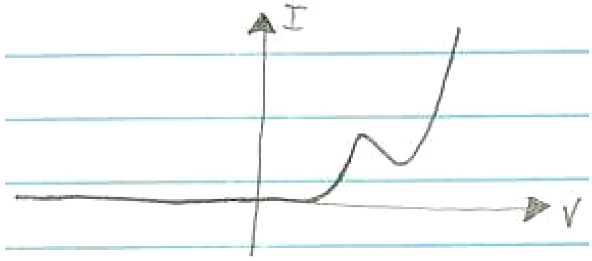

Figure 2.63:

2.7 Small Signal AC model of the Diode

Consider a (non-ideal) junction diode connected on a time-varying current source $I_D(t)$ as shown in figure 2.64(a). The current source $I_D(t)$ consists in a DC component, denoted as I_D and a *small* purely AC component[2] denoted as $i_D(t)$:

$$I_D(t) = I_D + i_D(t)$$

Figure 2.64(a) could be redrawn as in figure 2.64(b) to show both AC and DC

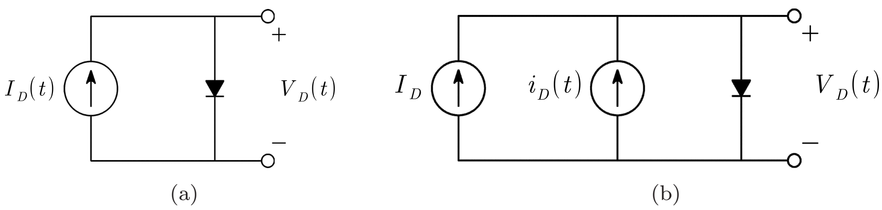

Figure 2.64:

components of $I_D(t)$. We next calculate $V_D(t)$ using the exponential characteristic of the diode (refer to section 2.3). Specifically, we will extract from the time-varying voltage $V_D(t)$ its DC component and its purely AC component, denoted respectively as V_D and $v_D(t)$:

$$V_D(t) = V_D + v_D(t)$$

From the general expression of the exponential characteristic

$$I = I_S(e^{V/(nV_T)} - 1)$$

[2]Not necessarily sinusoidal. $i_D(t)$ could be sine, triangular, square or any type of wave, the average value of which is 0.

we isolate the voltage V and replace I by $I_D(t)$ and V by $V_D(t)$. We then obtain:

$$\begin{aligned} V_D(t) &= nV_T \ln\left(\frac{I_D(t)}{I_S} + 1\right) \\ &= nV_T \ln\left(\left(\frac{I_D}{I_S} + 1\right) + \frac{i_D(t)}{I_S}\right) \end{aligned}$$

We then use the Taylor series expansion:

$$\ln\left(A + \frac{x}{B}\right) = \ln(A) + \frac{x}{BA} - \frac{x^2}{2B^2A^2} + \frac{x^3}{3B^3A^3} - \frac{x^4}{4B^4A^4} + \frac{x^5}{5B^5A^5} + \mathscr{O}\left(x^6\right)$$

in the neighbourhood of $x = 0$. If $\left|\frac{x}{AB}\right| \ll 1$ in the above, then we can approximate with the first two terms of the sum. Replacing:

$$\begin{aligned} A &= \frac{I_D}{I_S} + 1 \\ B &= I_S \\ x &= i_D(t) \end{aligned}$$

we obtain:

$$\begin{aligned} V_D(t) &\approx \underbrace{nV_T \ln\left(\frac{I_D}{I_S} + 1\right)}_{\text{DC component}} + \underbrace{\frac{nV_T}{\frac{I_D}{I_S} + 1}\frac{i_D(t)}{I_S}}_{\text{AC component}} \\ &= V_D + v_D(t) \end{aligned}$$

as long as:

$$\left|\frac{i_D(t)}{I_S\left(\frac{I_D}{I_S} + 1\right)}\right| \ll 1 \quad \Leftrightarrow \quad \left|i_D(t)\right| \ll I_D + I_S$$

and where

$$v_D(t) = \frac{nV_T}{I_D + I_S} i_D(t) \tag{2.10}$$

(the inequality $\left|i_D(t)\right| \ll I_D + I_S$ is equivalent to $\left|v_D(t)\right| \ll nV_T$). Equation (2.10) is of the form $V = R\,I$ and the factor $\frac{nV_T}{I_D+I_S}$ is called the *dynamic resistance* of the diode. It is easily seen, from the Taylor series expansion, to equal the multiplicative inverse of the slope of the tangent of the $I-V$ characteristic of the diode at the point $(V_D,\, I_D)$, as illustrated in figure 2.65. It follows from this analysis that the DC and AC components of $V_D(t)$ can be calculated separately. First the DC component is calculated using the exponential characteristic of the diode (or any of the piece-wise linear models presented in section 2.4) while setting the AC component(s) to 0. Next, the AC component is calculated by *modelling* the diode as a resistor of value

$$R_{\text{dynamic}} = \frac{nV_T}{I_D + I_S}$$

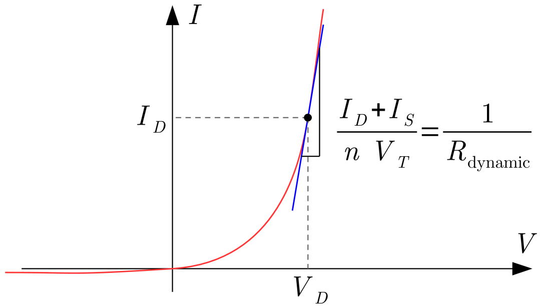

Figure 2.65:

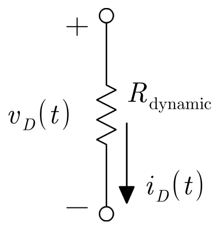

Figure 2.66: Small signal AC model of the junction diode

while setting the DC component(s) to 0. Finally, the DC and AC components are added to yield the approximate value of $V_D(t)$.

The procedure bears some resemblance with the application of the *superposition theorem* to the analysis of a linear circuit. It is conceptually important to remember that the superposition theorem does not apply to the present situation since the circuit is non-linear. What we have shown is that the superposition principle nonetheless applies as long as the AC components are relatively small and that the DC analysis is performed first in order to obtain the value I_D.

We can then say that from the point of view of small AC signals, a diode maybe represented by the equivalent linear circuit model of figure 2.66.

Example 2.7.1. To be seen in class ...

2.8 Problems

1. Calculate and sketch the V_o versus V_i input-output characteristic of each of the circuits in figure 2.67.

2. Exercises 3.22, 3.26 at page 210 in the Fourth Edition of Sedra & Smith.

3. Determine and sketch the $I-V$ characteristic of the circuits in figure 2.68