Solution : #1

(1) and (2)

$$S_y(f) = S_x(f)\,|H_y(f)|^2$$

$$= \begin{cases} S_x(f) & ; \ -2\text{kHz} \le f < 2\text{kHz} \\ 0 & ; \ \text{elsewhere} \end{cases}$$

$$\Rightarrow R_y(0) = \int_{-2k}^{2k} S_x(f)\,df$$

$$= \int_{-2k}^{2k} 0.001 \ \text{sinc}^2(f/1000)\,df \qquad f = 1000x$$

$$= 0.001 \int_{-2}^{2} \text{sinc}^2(x)\ 1000\ dx$$

$$= \int_{-2}^{2} \text{sinc}^2(x)\,dx$$

$$= \frac{1}{\pi^2(2)}\left(\cos(4\pi) - 1 + 4\pi\, Si(4\pi)\right)$$

$$= \frac{2\,Si(4\pi)}{\pi} = 0.94993934 \equiv \text{total average power of } y(t)$$

$$S_z(f) = S_x(f)\,|H_z(f)|^2$$

$$= \begin{cases} S_x(f) & ; \ |f| > 1\text{kHz} \\ 0 & ; \ \text{elsewhere} \end{cases}$$

$$\Rightarrow R_z(0) = \int_{|f|>1\text{kHz}} S_x(f)\,df$$

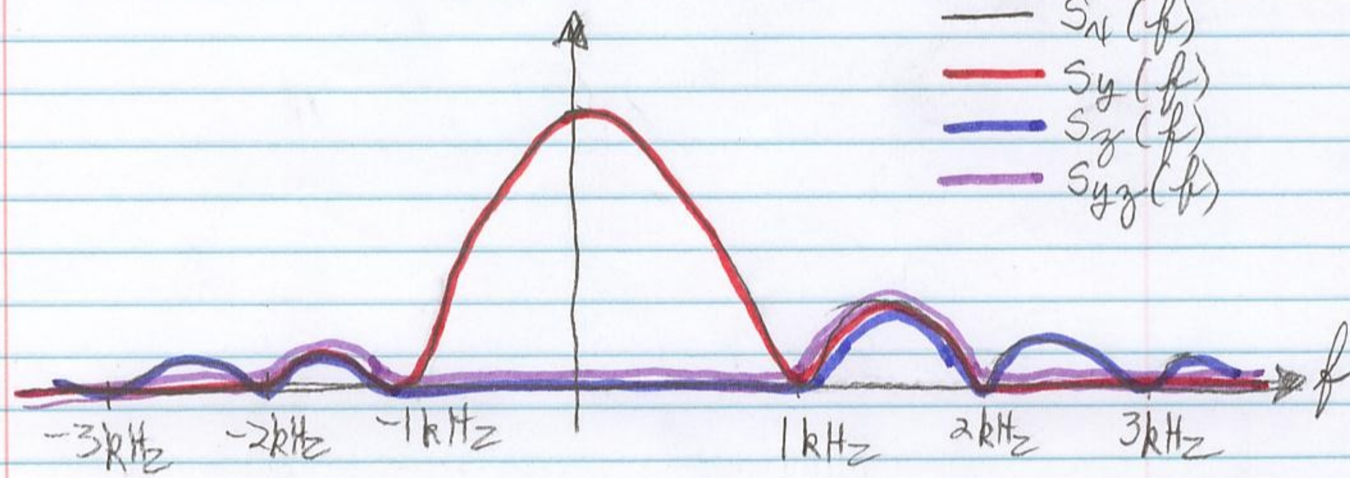
$$= \int_{|x|>1} \text{sinc}^2(x)\,dx$$

$$= \frac{1}{\pi^2}\left(1-\cos(2\pi) - 2\pi\, Si(2\pi) + \pi^2\right)$$

$$= \frac{\pi - 2\, Si(2\pi)}{\pi} = 0.097176666 \equiv \text{total average power of } z(t).$$

$$S_{yz}(f) = S_n(f)\, H_y(f)\, H_z^*(f)$$

$$= \begin{cases} S_n(f) & ; \; 1\,\text{kHz} < |f| < 2\,\text{kHz} \\ 0 & ; \; \text{elsewhere} \end{cases}$$



(3) $y(t), z(t)$ independent $\Rightarrow L_{yz}(\tau) = 0$

$= R_{yz}(\tau)$

because $m_y(t) = m_z(t) = 0$

Clearly $R_{yz}(\tau) \neq 0$ cuz $R_{yz}(0) = \int S_{yz}(f)\, df \neq 0$. This means that $y(t), z(t)$ are not statistically independent.

Solution **#2**

(1) $\bar{n}_1 = \overline{\int n_w(t)\, s_1(t)\, dt}$

$= \int \overline{n_w(t)}\, s_1(t)\, dt$

$= 0$

Similarly $\bar{n}_2 = 0 \Rightarrow \underline{m_n} = (0, 0)$.

$\overline{n_1^2} = Var(n_1)$ since $\bar{n}_1 = 0$

$= \overline{\iint n_w(t)\, s_1(t)\, n_w(s)\, s_1(s)\, dt\, ds}$

$= \iint 12\delta(t-s)\, s_1(t)\, s_1(s)\, ds\, dt$

$= \int 12\, s_1^2(t)\, dt$

$= 12\int_0^1 (1/2)^2\, dt = \frac{12}{4} = 3$

$\overline{n_2^2} = Var(n_2)$

$= \iint 12\delta(t-s)\, s_2(t)\, s_2(s)\, ds\, dt$

$= \int 12\, s_2^2(t)\, dt$

$= 12\int_0^1 t^2\, dt = \frac{12}{3} t^3 \Big|_0^1 = 4$

$\overline{n_1 n_2} = Cov(n_1, n_2)$

$= \iint 12\delta(t-s)\, s_1(t)\, s_2(s)\, ds\, dt$

$= 12\int s_1(t)\, s_2(t)\, dt$

$$= 12\int_0^1 \frac{t}{2}\, dt = \frac{12}{4} t^2 \Big|_0^1 = 3$$

$$\Lambda_n = \begin{bmatrix} 3 & 3 \\ 3 & 4 \end{bmatrix}$$

(2) n_1, n_2 are Gaussian $\Rightarrow$

$$n(1/2) = n_1 s_1(1/2) + n_2 s_2(1/2)$$

$$= \frac{n_1 + n_2}{2} \text{ is Gaussian.}$$

$$\overline{n(1/2)} = \frac{\overline{n_1} + \overline{n_2}}{2} = 0$$

$$Var(n(1/2)) = \begin{bmatrix} 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 3 & 3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

by lemma 29 in chapter 2.

$$Var(n(1/2)) = \begin{bmatrix} 3 & 7/2 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} = 3.25$$

$$P(n(1/2) > 1) = Q\left(\frac{1-0}{\sqrt{3.25}}\right) = 0.2895498709$$

from §2.3 in notes