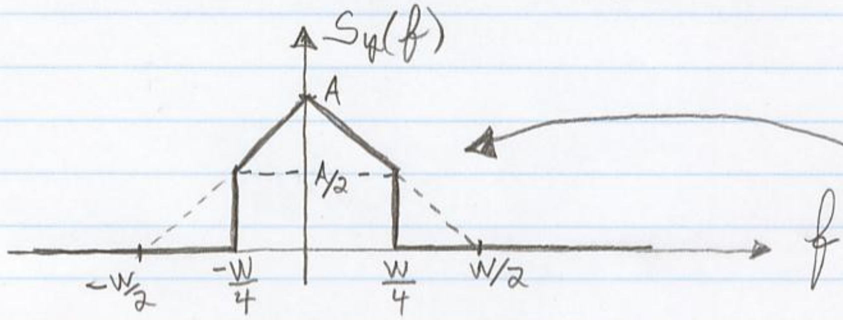
Solution #1:

(a) $m_y = m_z \int_{-\infty}^{\infty} h(t)\, dt$

$= m_z H(0) = 0$

$S_y(f) = S_z(f)\,|H(f)|^2$. Graphically we obtain



The expression for $S_y(f)$ is:

$$S_y(f) = \begin{cases} A + 2Af/W & ; -W/4 \le f \le 0 \\ A - 2Af/W & ; \ 0 \le f \le W/4 \\ 0 & ; \text{ elsewhere} \end{cases}$$

(b) total average power of $y(t)$ $\equiv R_y(0) = \int_{-\infty}^{\infty} S_y(f)\, df$ (area under)

$$= \frac{A}{2} \times \frac{W}{2} + \frac{1}{2} \times \frac{A}{2} \times \frac{W}{2}$$

$$= \frac{AW}{4} + \frac{AW}{8}$$

$$= \frac{3AW}{8}$$

Solution #2:

(1) $\overline{n_1} = \overline{\int n_w(t)\, s_1(t)\, dt}$

$= \int \overline{n_w(t)}\, s_1(t)\, dt$

$= 0$

Similarly $\overline{n_2} = 0 \Rightarrow \underline{m_n} = (0, 0)$.

$\overline{n_1^2} = \text{Var}(n_1)$ since $\overline{n_1} = 0$

$= \overline{\iint n_w(t)\, s_1(t)\, n_w(s)\, s_1(s)\, dt\, ds}$

$= \iint 12\delta(t-s)\, s_1(t)\, s_1(s)\, ds\, dt$

$= \int 12\, s_1^2(t)\, dt$

$= 12\int_0^1 (1/2)^2\, dt = \frac{12}{4} = 3$

$\overline{n_2^2} = \text{Var}(n_2)$

$= \iint 12\delta(t-s)\, s_2(t)\, s_2(s)\, ds\, dt$

$= \int 12\, s_2^2(t)\, dt$

$= 12\int_0^1 t^2\, dt = \frac{12}{3} t^3 \Big|_0^1 = 4$

$\overline{n_1 n_2} = \text{Cov}(n_1, n_2)$

$= \iint 12\delta(t-s)\, s_1(t)\, s_2(s)\, ds\, dt$

$= 12 \int s_1(t)\, s_2(t)\, dt$

$$= 12\int_0^1 \frac{t}{2}\, dt = \frac{12}{4}\, t^2\Big|_0^1 = 3$$

$$\Lambda_n = \begin{bmatrix} 3 & 3 \\ 3 & 4 \end{bmatrix}$$

(2) n_1, n_2 are Gaussian $\Rightarrow$

$$n(1/2) = n_1\, s_1(1/2) + n_2\, s_2(1/2)$$

$$= \frac{n_1 + n_2}{2} \text{ is Gaussian.}$$

$$\overline{n(1/2)} = \frac{\overline{n_1} + \overline{n_2}}{2} = 0$$

$$Var(n(1/2)) = \begin{bmatrix} 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 3 & 3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

by lemma 29 in chapter 2.

$$Var(n(1/2)) = \begin{bmatrix} 3 & 7/2 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} = 3.25$$

$$P(n(1/2) > 1) = Q\left(\frac{1-0}{\sqrt{3.25}}\right) = 0.2895498709$$

from §2.3 in notes

$\boxed{\#2}$

$$R_n(t,s) = E\left[n(t)\, n(s)\right]$$

$$= E\left[\left(n_1 s_1(t) + n_2 s_2(t)\right)\left(n_1 s_1(s) + n_2 s_2(s)\right)\right]$$

$\Lambda_{(n_1,n_2)} = \begin{bmatrix} 3 & 3 \\ 3 & 4 \end{bmatrix}$

$$= E\left[n_1^2 s_1(t) s_1(s)\right] + E\left[n_1 s_1(t) n_2 s_2(s)\right]$$

$$+ E\left[n_2 s_2(t) n_1 s_1(s)\right] + E\left[n_2^2 s_2(t) s_2(s)\right]$$

$$= 3 s_1(t) s_1(s) + 3 s_1(t) s_2(s) + 3 s_1(s) s_2(t)$$

$$+ 4 s_2(t) s_2(s)$$

if $0<t<1$, $0<s<1$

$$= \frac{3}{4} + \frac{3s}{2} + \frac{3}{2}t + 4ts$$

$$= \begin{cases} \left(3 + 6(s+t) + 16ts\right)/4 \ ; & 0<t,s<1 \\ 0 \ ; & \text{elsewhere} \end{cases}$$

$$m_n(t) = E\left[n(t)\right]$$

$$= \bar{n}_1 s_1(t) + \bar{n}_2 s_2(t)$$

$$= 0$$

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at $t = 1/2$ $\quad n(1/2)$ is Gauss 0-mean with variance $R_n(1/2, 1/2) = 3.25$