Solution # 1

We rewrite $p_X(\alpha)$ as

$$p_X(\alpha) = \frac{1}{\sqrt{2\pi \cdot 12.5}} e^{-(\alpha-2)^2/(2\cdot 12.5)}$$
$$= \frac{1}{\sqrt{2\pi \sigma^2}} e^{-(\alpha-\mu)^2/(2\sigma^2)}$$

$$\Rightarrow \mu = 2 \text{ and } \sigma^2 = 12.5$$

(a) $P(X < -4) = Q\left(\frac{2+4}{\sqrt{12.5}}\right) \doteq 0.044843\cdots$

(b) $P(X \geq 4) = Q\left(\frac{4-2}{\sqrt{12.5}}\right) = 0.28580\cdots$

(c) $P(-1 \leq X < 3) = Q\left(\frac{-1-2}{\sqrt{12.5}}\right) - Q\left(\frac{3-2}{\sqrt{12.5}}\right)$
$$= 1 - Q\left(\frac{3}{\sqrt{12.5}}\right) - Q\left(\frac{1}{\sqrt{12.5}}\right)$$
$$= 0.413279\cdots$$

Solution # 2

(a) $P(A) = \int_{-\infty}^{\infty} p_N(\alpha, A)\, d\alpha$

$= \int_{0}^{\infty} \frac{1}{4} e^{-\alpha}\, d\alpha$

$= \frac{1}{4} \left[-e^{-\alpha}\right]_0^{\infty} = 1/4$

$P(B) = \int_{-\infty}^{\infty} p_N(\alpha, B)\, d\alpha$

$= \int_0^1 1/2\, d\alpha = 1/2$

$P(A \cup B) = P(A) + P(B) = 1/4 + 1/2 = 3/4$

cuz $A \cap B = \emptyset$

(b) $P(\{N \geq 1/2\} | A) = \int_{1/2}^{\infty} p_N(\alpha | A)\, d\alpha$

$= \int_{1/2}^{\infty} \frac{p_N(\alpha, A)}{P(A)}\, d\alpha$

$= 4 \int_{1/2}^{\infty} \frac{1}{4} e^{-\alpha}\, d\alpha$

$= \left[-e^{-\alpha}\right]_{1/2}^{\infty} = e^{-1/2} = 1/\sqrt{e}$

(c) $P(A | N = 1/2) = \frac{p_N(1/2, A)}{p_N(1/2)} = \frac{16}{11 + 4e^{-1/2}} \times \frac{e^{-1/2}}{4} = \frac{4}{11\sqrt{e} + 4}$

$\approx 0.1807017.$

Solution: # 3

(a) $p_y(\beta) = \int_{-\infty}^{\infty} p_{xy}(\alpha, \beta)\, d\alpha$

$$= \int_{\beta-3}^{\beta+3} \frac{9-(\alpha-\beta)^2}{36}\, d\alpha \qquad 0 \le \beta < 1$$

$$= \frac{1}{36}\left[9\alpha - \frac{(\alpha-\beta)^3}{3}\right]_{\beta-3}^{\beta+3} \qquad 0 \le \beta < 1$$

$$= \frac{1}{36}\left(9(\beta+3) - \frac{3^3}{3} - 9(\beta-3) + \frac{(-3)^3}{3}\right) \qquad 0 \le \beta < 1$$

$$= \frac{27 - 9 + 27 - 9}{36} \qquad 0 \le \beta < 1$$

$$= \begin{cases} 1 & ; \ 0 \le \beta < 1 \\ 0 & ; \ \text{elsewhere} \end{cases}$$

(b) $p_x(\alpha | y = 3/4) = \frac{p_{xy}(\alpha, 3/4)}{p_y(3/4)} = \frac{p_{xy}(\alpha, 3/4)}{1}$

$$= \begin{cases} \frac{9-(\alpha-3/4)^2}{36} & ; \ -2.25 \le \alpha < 3.75 \\ 0 & ; \ \text{elsewhere} \end{cases}$$

$$P(x \ge 3.5 | y = 3/4) = \int_{3.5}^{\infty} p_x(\alpha | y = 3/4)\, d\alpha$$

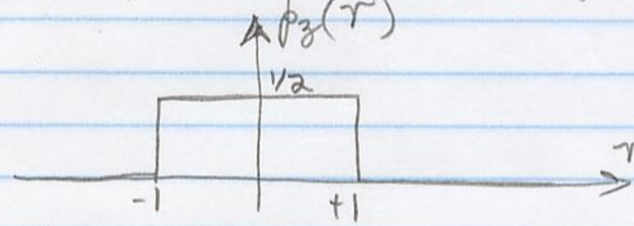
$$= \int_{3.5}^{3.75} \frac{9-(\alpha-3/4)^2}{36}\, d\alpha$$

$$= \frac{1}{36}\left[9\alpha - \frac{(\alpha - 3/4)^3}{3}\right]_{3.5}^{3.75}$$

$$= \frac{9\cdot 3.75 - 3^3/3 - 9\cdot 3.5 + 2.75^3/3}{36}$$

$$= 5.063657 \times 10^{-3}$$

(c) Clearly z is uniform from -1 to $+1$:



More formally:

$$\begin{aligned} F_z(r) &= P(z \le r) \\ &= P(1-2y \le r) \\ &= P(y \ge (1-r)/2) \\ &= 1 - P(y < (1-r)/2) \\ &= 1 - F_y\left(\frac{1-r}{2}\right) \end{aligned}$$

$$\Rightarrow p_z(r) = -p_y\left(\frac{1-r}{2}\right) \times \left(\frac{-1}{2}\right) = \frac{p_y((1-r)/2)}{2}$$

$$= \begin{cases} 1/2 \ ; & 0 \le \frac{1-r}{2} < 1 \Leftrightarrow -1 < r \le 1 \\ 0 \ ; & \text{elsewhere} \end{cases}$$