Solution # 1:

Refer to the solution of problem 2.7 in W&J.

Solution # 2

(a) $P(TX = m_1 | RX = 0) = \dfrac{P(RX=0 | TX = m_1)\, P(m_1)}{P(RX=0)}$

où

$$P(RX=0 | TX = m_1) = 0.01$$
$$P(m_1) = 2/3$$
$$\begin{aligned} P(RX=0) &= P(RX=0|TX=m_0)\,P(m_0) \\ &\quad + P(RX=0|TX=m_1)\,P(m_1) \\ &= 0.99 \cdot 1/3 + 0.01 \cdot 2/3 \\ &= 101/300 \approx 0.33666\ldots \end{aligned}$$

On obtient alors:

$$P(TX=m_1 | RX=0) = \frac{0.01 \cdot 2/3}{101/300} = 2/101 \approx 0.019802$$

(b) $P(TX = m_0 | RX = 1) = \dfrac{P(RX=1 | TX=m_0)\, P(m_0)}{P(RX=1)}$

où

$$P(RX=1 | TX=m_0) = 0.01$$
$$P(m_0) = 1/3$$
$$\begin{aligned} P(RX=1) &= 1 - P(RX=0) \\ &= 199/300 \approx 0.66333\ldots \end{aligned}$$

On obtient alors

$$P(TX=m_0 | RX=1) = \frac{0.01 \cdot 1/3}{199/300} = 1/199 \approx 0.0050251$$

(c) $$\begin{aligned} P(\text{erreur}) &= P(\text{erreur} | RX=0)\,P(RX=0) \\ &\quad + P(\text{erreur}|RX=1)\,P(RX=1) \\ &= \frac{2}{101} \cdot \frac{101}{300} + \frac{1}{199} \cdot \frac{199}{300} = 1/100 = 0.01 \end{aligned}$$

$$\begin{aligned} P(\text{erreur}) &= P(\text{erreur} | TX=m_0)\,P(TX=m_0) \\ &\quad + P(\text{erreur} | TX=m_1)\,P(TX=m_1) \\ &= \frac{1}{100} \cdot \frac{1}{3} + \frac{1}{100} \cdot \frac{2}{3} = 1/100 = 0.01 \end{aligned}$$

Aussi voir théorème 10.2.1 dans les notes

Solution # 3

$F_Y(\alpha)$ is given by

$$F_Y(\alpha) = \int p_Y(\alpha)\, d\alpha$$

$$= \begin{cases} \int 0\, d\alpha = K_1 \;; & \boxed{\alpha < 3} \\ \int \dfrac{9-\alpha^2}{36}\, d\alpha = \dfrac{9\alpha - \alpha^3/3}{36} + K_2 \;; & \boxed{-3 \leq \alpha < 3} \\ \int 0\, d\alpha = K_3 \;; & \boxed{\alpha \geq 3} \end{cases}$$

Constants K_1, K_2, K_3 are found using the properties of the distribution function; monotonically increasing function from 0 at $-\infty$ to $+1$ at $+\infty$. Moreover because $p_Y(\alpha)$ has no impulses, $F_Y(\alpha)$ has no steps. We then find

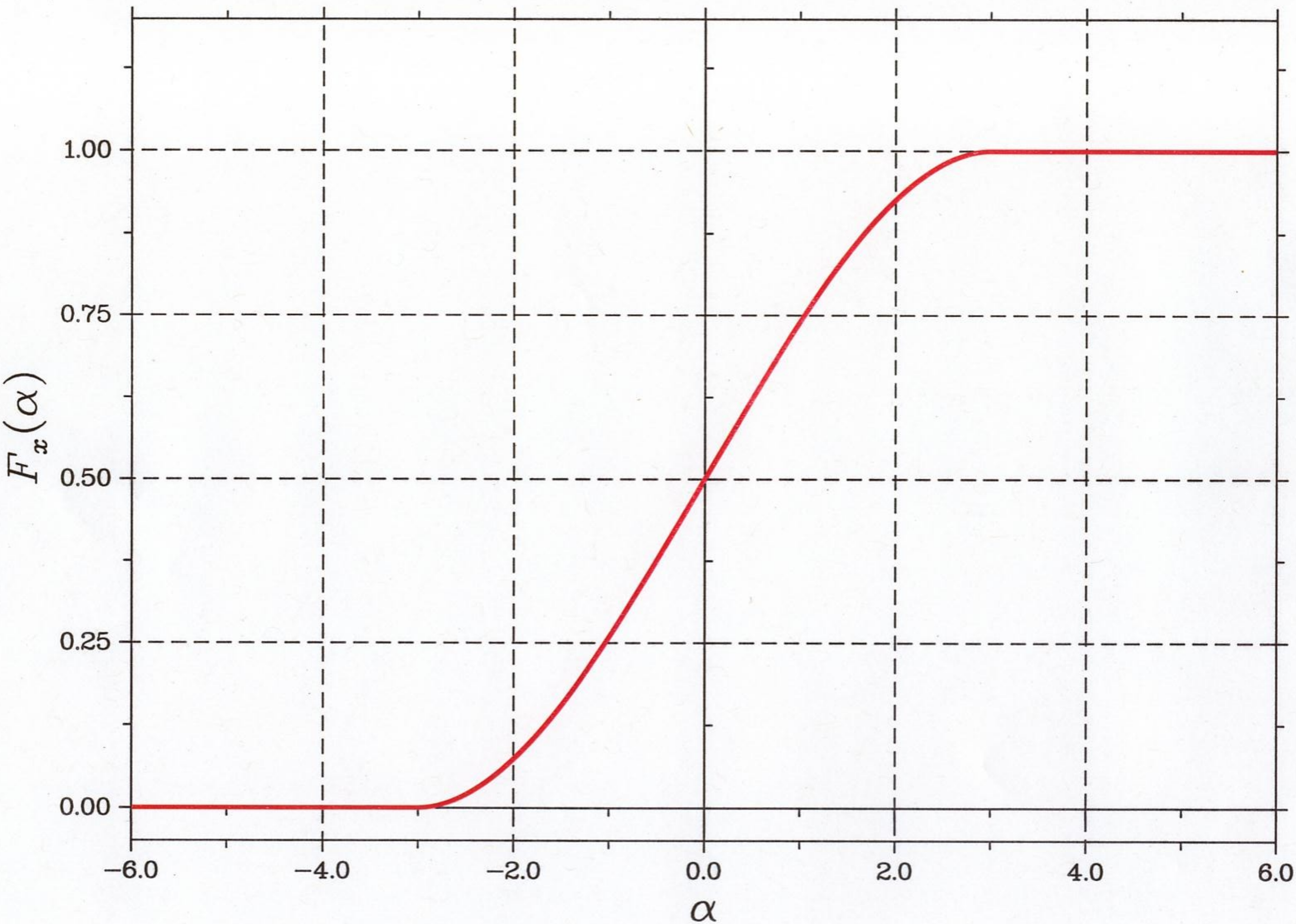
- $K_1 = 0$ because $F_Y(-\infty) = K_1$
- $K_3 = 1$ because $F_Y(+\infty) = K_3$
- $K_2 = 0.5$ because

$$\left.\begin{aligned} F_Y(-3) &= -0.5 + K_2 = K_1 = 0 \\ F_Y(3) &= 0.5 + K_2 = K_3 = 1 \end{aligned}\right\} \Rightarrow K_2 = 0.5$$

In summary

$$F_Y(\alpha) = \begin{cases} 0 & \alpha < -3 \\ 1/2 + \alpha/4 - \alpha^3/108 & -3 \leq \alpha < 3 \\ 1 & \alpha \geq 3 \end{cases}$$

See next page for a graph.



$F_x(\alpha)$
1.00
0.75
0.50
0.25
0.00
−6.0
−4.0
−2.0
0.0
2.0
4.0
6.0
α