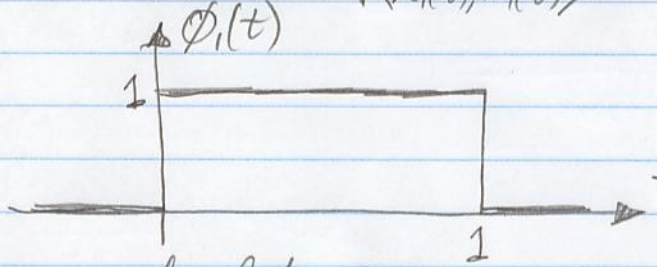
Solution Problem #1, section 4.6:

(a) $\langle s_1(t), s_1(t)\rangle = \int_0^1 s_1^2(t)\,dt = \frac{1}{4}\int_0^1 dt = 1/4$. We define $\phi_1(t) = \frac{s_1(t)}{\sqrt{\langle s_1(t), s_1(t)\rangle}} = 1,\ 0 \le t \le 1$:



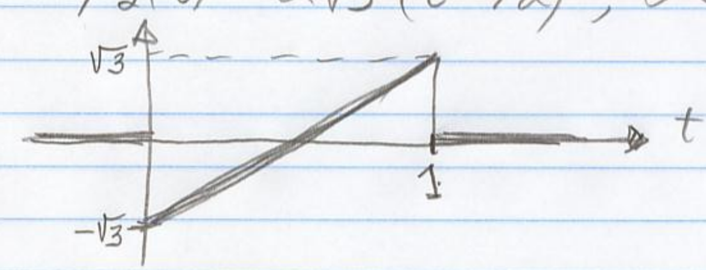
Next we calculate $\theta_2(t) = s_2(t) - \langle s_2(t), \phi_1(t)\rangle \phi_1(t)$:

$$\langle s_2(t), \phi_1(t)\rangle = \int_0^1 t\,dt = 1/2$$

$$\Rightarrow \theta_2(t) = t - 1/2,\quad 0 \le t \le 1.$$

We find $\langle \theta_2(t), \theta_2(t)\rangle = \int_0^1 (t - 1/2)^2\,dt = 1/12 \Rightarrow$

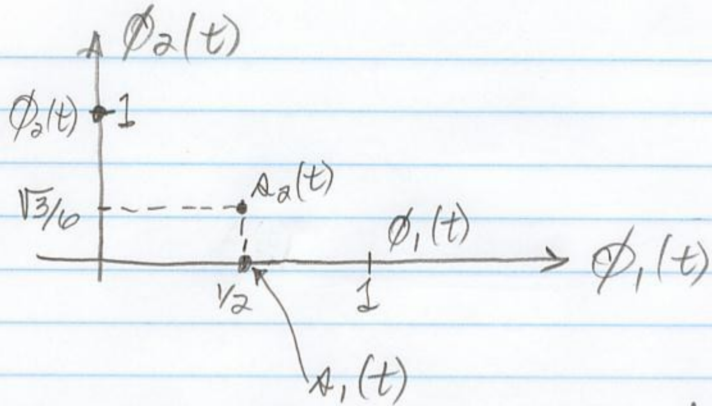
$$\phi_2(t) = 2\sqrt{3}\,(t - 1/2),\quad 0 \le t \le 1$$



(b)
$$\left.\begin{aligned} \langle s_1(t), \phi_1(t)\rangle &= 1/2 \\ \langle s_1(t), \phi_2(t)\rangle &= 0 \end{aligned}\right\} s_1(t) \longleftrightarrow (1/2, 0) = \underline{s}_1$$
$$\left.\begin{aligned} \langle s_2(t), \phi_1(t)\rangle &= 1/2 \\ \langle s_2(t), \phi_2(t)\rangle &= \sqrt{3}/6 \end{aligned}\right\} s_2(t) \longleftrightarrow (1/2, \sqrt{3}/6) = \underline{s}_2$$

easy to calculate

The functions/signals $s_1(t)$, $s_2(t)$ are represented by points in a 2-dimensional cartesian plane:



(c) We calculate the "distance" between $s_1(t)$ and $\phi_1(t)$, $\phi_2(t)$

$$\sqrt{\langle s_1(t)-\phi_1(t), s_1(t)-\phi_1(t)\rangle} = 1/2 \quad \leftarrow \text{closer to } \phi_1(t)$$

$$\sqrt{\langle s_1(t)-\phi_2(t), s_1(t)-\phi_2(t)\rangle} = \sqrt{5}/2 \approx 1.118$$

$\Rightarrow s_1(t)$ resemble more $\phi_1(t)$ than $\phi_2(t)$.

Similarly for $s_2(t)$:

$$\sqrt{\langle s_2(t)-\phi_1(t), s_2(t)-\phi_1(t)\rangle} = \sqrt{3}/3 \approx 0.577$$

$$\sqrt{\langle s_2(t)-\phi_2(t), s_2(t)-\phi_2(t)\rangle} = \frac{\sqrt{12-3\sqrt{3}}}{3} \approx 0.869$$

closer to $\phi_1(t)$

$\Rightarrow s_2(t)$ resemble more $\phi_1(t)$ than $\phi_2(t)$.

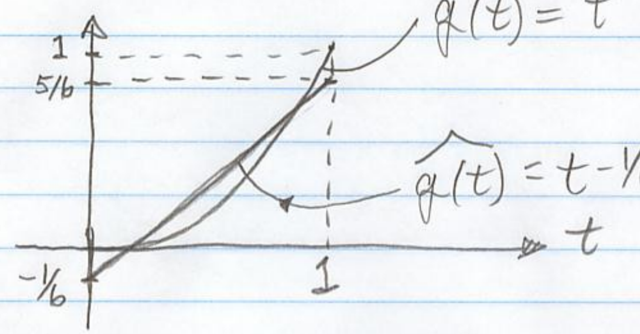
(d) Simply project $g(t)$ on the basis functions:

$$\langle g(t), \phi_1(t)\rangle = \int_0^1 t^2\, dt = 1/3$$

$$\langle g(t), \phi_2(t)\rangle = \int_0^1 (t^2)(2\sqrt{3})(t-1/2)\, dt = \sqrt{3}/6$$

$$\Rightarrow \widehat{g(t)} = 1/3\, \phi_1(t) + \frac{\sqrt{3}}{6}\phi_2(t) = t - 1/6$$

$g(t)$ and $\widehat{g(t)}$ are sketched versus t:



The result appear to make sense as $\widehat{g(t)}$ appears to be to closest straight line segment to approximate the parabola.

(e) Calculate

$$\theta_3(t) = g(t) - \langle g(t), \phi_1(t) \rangle \phi_1(t) - \langle g(t), \phi_2(t) \rangle \phi_2(t)$$

where

$$\langle g(t), \phi_1(t) \rangle = 1/3$$

$$\langle g(t), \phi_2(t) \rangle = \sqrt{3}/6$$

We then obtain

$$\theta_3(t) = t^2 - t + 1/6 \quad , 0 \le t \le 1$$

$$\langle \theta_3(t), \theta_3(t) \rangle = 1/180$$

Finaly we define

$$\phi_3(t) = \frac{\theta_3(t)}{\sqrt{1/180}} = \sqrt{5}\left(6t^2 - 6t + 1\right)$$

$\{\phi_1(t), \phi_2(t), \phi_3(t)\}$ form an orthanormal basis in which $s_1(t)$, $s_2(t)$, $g(t)$ can be represented:

$$s_1(t) \longleftrightarrow (1/2, 0, 0)$$

$$s_2(t) \longleftrightarrow (1/2, \sqrt{3}/6, 0)$$

$$g(t) \longleftrightarrow (1/3, \sqrt{3}/6, \sqrt{5}/30)$$