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EE501: An Introduction to the Theory of Statistical Communications

Friday, 7 December 2018

Fourth Quiz

REMARKS:

1. Hand held calculator is allowed,
2. Open book quiz, but problem solutions are not allowed,
3. A table of the $Q(\ )$ and $\mathrm{erf}(\ )$ functions is attached,
4. A table of the $\mathrm{Si}(\ )$ function is attached,
5. Marks distribution:
   Question #1: 5 points
   Question #2: 5 points
6. Justify all your answers.

# 1	
# 2	

1. Let $x(t)$ denote the stationary Binary Random Process of example 3.3.5. With a rectangular pulse of duration $T = 1$ ms. We have found that

$$m_x(t) = 0$$

$$\mathscr{R}_x(\tau) = \begin{cases} 1 - 1000\,|\tau| & ; \quad -1 \text{ ms} < \tau < 1 \text{ ms} \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

$$S_x(f) = 0.001\ \mathrm{sinc}^2(f/1000).$$

$S_x(f)$ was also sketched in figure 3.12. $x(t)$ is fed through 2 filters as shown in figure 1. The filters frequency responses are given by:

$$H_y(f) = \begin{cases} 1 & ; \quad -2 \text{ kHz} < f < 2 \text{ kHz} \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

$$H_z(f) = \begin{cases} 1 & ; \quad |f| > 1 \text{ kHz} \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

$x(t)$, $y(t)$, $z(t)$ are all jointly stationary non-Gaussian random processes.

(a) Calculate and/or sketch $S_y(f)$, $S_z(f)$, $S_{yz}(f)$.

(b) Calculate the total average power of $y(t)$ and $z(t)$.
Hint: You need not calculate $\mathscr{R}_y(\tau)$, $\mathscr{R}_z(\tau)$.

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(c) Are the processes $y(t)$, $z(t)$ statistically independent? Justify your answer.

Note: The following may be useful ($a > 0$):

$$\int_{-a}^{a} \mathrm{sinc}^2(x)\mathrm{d}x = \frac{\cos(2\pi a) - 1 + 2\pi a\,\mathrm{Si}(2\pi a)}{\pi^2 a}$$

$$\int_{|x|>a} \mathrm{sinc}^2(x)\mathrm{d}x = \frac{\pi^2 a + 1 - \cos(2\pi a) - 2\pi a\,\mathrm{Si}(2\pi a)}{\pi^2 a}$$

and a table of the function Si( ) is attached.

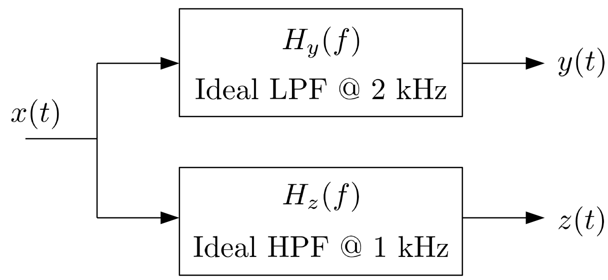

Figure 1:

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2. Let $n_w(t)$ be a 0-mean stationary white Gaussian process with power spectral density $S_{n_w}(f) = 12$, $\forall f$ and autocorrelation function $\mathscr{R}_{n_w}(\tau) = 12\delta(\tau)$. We define the random process

$$n(t) = n_1 s_1(t) + n_2 s_2(t)$$

where n_1, n_2 are the random variables given by:

$$n_1 = \int_{-\infty}^{\infty} n_w(t) s_1(t) \mathrm{d}t$$
$$n_2 = \int_{-\infty}^{\infty} n_w(t) s_2(t) \mathrm{d}t$$

and

$$s_1(t) = \begin{cases} 1/2 & ; \quad 0 < t < 1 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$
$$s_2(t) = \begin{cases} t & ; \quad 0 < t < 1 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

as sketched in figure 2.

(a) Calculate the mean vector $\boldsymbol{m_n}$ and covariance matrix $\Lambda_{\boldsymbol{n}}$ of the random vector $\boldsymbol{n} = (n_1, n_2)$.
Hint: You may interchange expectations and integrals.

(b) Calculate the probability that the random process $n(t)$ be larger than 1 at $t = 1/2$, i.e. $P(n(1/2) > 1)$.

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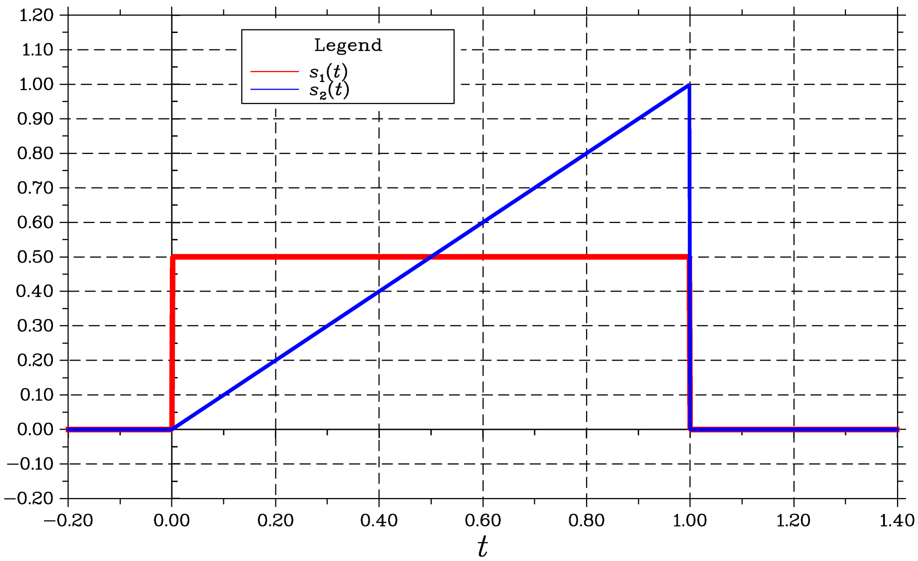

Figure 2:

END

Table of the $Q(\quad)$ and $\mathrm{erf}(\quad)$ functions

The approximation $Q(x) \approx \frac{1}{x\sqrt{2\pi}}(1 - \frac{0.7}{x^2})e^{-x^2/2}$ may be used when $x > 2$.

x	$\mathrm{erf}(x)$	$Q(x)$	x	$\mathrm{erf}(x)$	$Q(x)$	x	$\mathrm{erf}(x)$	$Q(x)$
0.00	0	0.5	1.70	0.9838	0.04457	3.40	1	0.0003369
0.10	0.1125	0.4602	1.80	0.9891	0.03593	3.50	1	0.0002326
0.20	0.2227	0.4207	1.90	0.9928	0.02872	3.60	1	0.0001591
0.30	0.3286	0.3821	2.00	0.9953	0.02275	3.70	1	0.0001078
0.40	0.4284	0.3446	2.10	0.997	0.01786	3.80	1	7.235×10^{-5}
0.50	0.5205	0.3085	2.20	0.9981	0.0139	3.90	1	4.810×10^{-5}
0.60	0.6039	0.2743	2.30	0.9989	0.01072	4.00	1	3.167×10^{-5}
0.70	0.6778	0.242	2.40	0.9993	0.008198	4.10	1	2.066×10^{-5}
0.80	0.7421	0.2119	2.50	0.9996	0.00621	4.20	1	1.335×10^{-5}
0.90	0.7969	0.1841	2.60	0.9998	0.004661	4.30	1	8.540×10^{-6}
1.00	0.8427	0.1587	2.70	0.9999	0.003467	4.40	1	5.413×10^{-6}
1.10	0.8802	0.1357	2.80	0.9999	0.002555	4.50	1	3.398×10^{-6}
1.20	0.9103	0.1151	2.90	1	0.001866	4.60	1	2.112×10^{-6}
1.30	0.934	0.0968	3.00	1	0.00135	4.70	1	1.301×10^{-6}
1.40	0.9523	0.08076	3.10	1	0.0009676	4.80	1	7.933×10^{-7}
1.50	0.9661	0.06681	3.20	1	0.0006871	4.90	1	4.792×10^{-7}
1.60	0.9763	0.0548	3.30	1	0.0004834	5.00	1	2.867×10^{-7}

Table of the Si( ) **function:** $\mathrm{Si}(-x) = -\mathrm{Si}(x)$, i.e. the function is odd.

x	$\mathrm{Si}(2\pi x)$	x	$\mathrm{Si}(2\pi x)$	x	$\mathrm{Si}(2\pi x)$
0.00000	0.00000	1.00000	1.41815	2.00000	1.49216
0.05000	0.31244	1.05000	1.42569	2.05000	1.49599
0.10000	0.61470	1.10000	1.44667	2.10000	1.50687
0.15000	0.89719	1.15000	1.47794	2.15000	1.52343
0.20000	1.15148	1.20000	1.51568	2.20000	1.54383
0.25000	1.37076	1.25000	1.55583	2.25000	1.56593
0.30000	1.55023	1.30000	1.59442	2.30000	1.58755
0.35000	1.68730	1.35000	1.62792	2.35000	1.60664
0.40000	1.78166	1.40000	1.65356	2.40000	1.62147
0.45000	1.83524	1.45000	1.66945	2.45000	1.63081
0.50000	1.85194	1.50000	1.67476	2.50000	1.63396
0.55000	1.83732	1.55000	1.66968	2.55000	1.63089
0.60000	1.79816	1.60000	1.65535	2.60000	1.62212
0.65000	1.74191	1.65000	1.63370	2.65000	1.60871
0.70000	1.67622	1.70000	1.60722	2.70000	1.59213
0.75000	1.60837	1.75000	1.57871	2.75000	1.57408
0.80000	1.54487	1.80000	1.55100	2.80000	1.55636
0.85000	1.49104	1.85000	1.52667	2.85000	1.54065
0.90000	1.45072	1.90000	1.50788	2.90000	1.52840
0.95000	1.42621	1.95000	1.49612	2.95000	1.52066
1.00000	1.41815	2.00000	1.49216	3.00000	1.51803