Name: ____________________ College Number: ____________________

EE501: An Introduction to the Theory of Statistical Communications

Thursday, 18 October 2018

Third Quiz

REMARKS:

1. Hand held calculator is allowed,
2. Open book quiz, but problem solutions are not allowed,
3. A table of the $Q(\ )$ and erf( ) functions is attached,
4. Marks distribution:
   Question #1: 3 points
   Question #2: 7 points
5. Justify all your answers.

# 1	
# 2	

1. Let x_1, x_2, ..., x_{1000} denote 1000 statistically independent identically distributed random variables with mean and variance respectively denoted by $\bar{x}$, σ_x^2. x_i takes the values -1, 0, 1 with probabilities:

$$\begin{aligned} P(x_i = -1) &= 1/2, \\ P(x_i = 0) &= 1/4, \\ P(x_i = 1) &= 1/4, \end{aligned}$$

for every $i = 1, 2, \ldots, 1000$. The sample mean m of x_1, x_2, ..., x_{1000} is defined by:

$$m = \frac{1}{1000} \sum_{i=1}^{1000} x_i$$

   (a) Using the Weak Law of Large Numbers, estimate the range within which m lies with a probability of at least 95%. In other words, use the Weak Law of Large Numbers to find a, b such that $P(a < m < b) \geq 0.95$.
   Remark: You can also do this with Chebyshev's inequality and find the same answer.

   (b) Using the expression:

$$P\left(\left|m - \bar{x}\right| \geq \epsilon\right) \approx 2Q\left(\frac{\epsilon\sqrt{N}}{\sigma_x}\right)$$

   obtained from the Central Limit Theorem, estimate the range within which m lies with a probability of approximately 95%.

Name: ____________________ College Number: ____________________

2. Consider the random process defined by $x(t) = 10\sin(2\pi f t)$ where f is a random variable uniformly distributed between 0 Hz and 100 Hz. This is very similar to the random process of equations 3.24(a) and 3.24(b) on page 143 in Wozencraft & Jacobs. The process is not stationary.

   (a) Calculate the mean function $m_x(t)$ and the autocorrelation function $\mathscr{R}_x(t_1, t_2)$ of $x(t)$ and show that:

$$m_x(t) = \frac{1 - \cos(200\pi t)}{20\pi t}$$

$$\mathscr{R}_x(t_1, t_2) = 50\Big(\mathrm{sinc}\big(200(t_1 - t_2)\big) - \mathrm{sinc}\big(200(t_1 + t_2)\big)\Big)$$

   Is the process wide sense stationary? Justify your answer.

   (b) Calculate the variance of the random variable $z = 2x(1\text{ ms}) - x(2\text{ ms})$.

END

Table of the $Q(\quad)$ and $\mathrm{erf}(\quad)$ functions

The approximation $Q(x) \approx \frac{1}{x\sqrt{2\pi}}(1 - \frac{0.7}{x^2})e^{-x^2/2}$ may be used when $x > 2$.

x	$\mathrm{erf}(x)$	$Q(x)$	x	$\mathrm{erf}(x)$	$Q(x)$	x	$\mathrm{erf}(x)$	$Q(x)$
0.00	0	0.5	1.70	0.9838	0.04457	3.40	1	0.0003369
0.10	0.1125	0.4602	1.80	0.9891	0.03593	3.50	1	0.0002326
0.20	0.2227	0.4207	1.90	0.9928	0.02872	3.60	1	0.0001591
0.30	0.3286	0.3821	2.00	0.9953	0.02275	3.70	1	0.0001078
0.40	0.4284	0.3446	2.10	0.997	0.01786	3.80	1	7.235×10^{-5}
0.50	0.5205	0.3085	2.20	0.9981	0.0139	3.90	1	4.810×10^{-5}
0.60	0.6039	0.2743	2.30	0.9989	0.01072	4.00	1	3.167×10^{-5}
0.70	0.6778	0.242	2.40	0.9993	0.008198	4.10	1	2.066×10^{-5}
0.80	0.7421	0.2119	2.50	0.9996	0.00621	4.20	1	1.335×10^{-5}
0.90	0.7969	0.1841	2.60	0.9998	0.004661	4.30	1	8.540×10^{-6}
1.00	0.8427	0.1587	2.70	0.9999	0.003467	4.40	1	5.413×10^{-6}
1.10	0.8802	0.1357	2.80	0.9999	0.002555	4.50	1	3.398×10^{-6}
1.20	0.9103	0.1151	2.90	1	0.001866	4.60	1	2.112×10^{-6}
1.30	0.934	0.0968	3.00	1	0.00135	4.70	1	1.301×10^{-6}
1.40	0.9523	0.08076	3.10	1	0.0009676	4.80	1	7.933×10^{-7}
1.50	0.9661	0.06681	3.20	1	0.0006871	4.90	1	4.792×10^{-7}
1.60	0.9763	0.0548	3.30	1	0.0004834	5.00	1	2.867×10^{-7}