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EE501: An Introduction to the Theory of Statistical Communications

Tuesday, 16 October 2018

Second Quiz

REMARKS:

1. Hand held calculator is allowed,
2. Open book quiz, but problem solutions are not allowed,
3. Answer any two of the three questions,
4. A table of the $Q(\ )$ and $\mathrm{erf}(\ )$ functions is attached,
5. Marks distribution:
   Question #1: 5 points
   Question #2: 5 points
   Question #3: 5 points
6. Justify all your answers.

# 1	
# 2	
# 3	

1. Let y denote a random variable defined by the transformation

$$y = \sqrt{-2\ln(x)}\,,$$

where x is a random variable uniformly distributed between 0 and 1. Calculate the probability density function $p_y(\beta)$ of y and confirm that y is a Rayleigh distributed random variable.

Suggestion: You may use any of the three techniques described in sections 2.4.1, 2.4.2, 2.4.3.

Recall:

$$\begin{aligned}
\frac{\mathrm{d}\sqrt{-2\ln(u)}}{\mathrm{d}u} &= \frac{-1}{u\sqrt{-2\ln(u)}} \\
\frac{\mathrm{d}\ \mathrm{e}^{u}}{\mathrm{d}u} &= \mathrm{e}^{u} \\
\frac{\mathrm{d}\ \mathrm{e}^{-u^2/2}}{\mathrm{d}u} &= -u\mathrm{e}^{-u^2/2}
\end{aligned}$$

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2. **Subquestion (a) only of the following (problem 2.25 in Wozencraft & Jacobs):**
A "diversity" communication system employs two channels to transmit a voltage s to a decision device as shown in figure 1. Thus the decision device has available two received voltages, r_1 and r_2, in which

$$r_1 = s + n_1 \,, \qquad\qquad r_2 = s + n_2 \,.$$

Assume that n_1 and n_2 are zero-mean Gaussian random variables with variances σ_1^2 and σ_2^2 and that s, n_1, and n_2 are jointly statistically independent. The system is used to communicate one of two messages m_0 and m_1 with a priori probabilities $P[m_0]$ and $P[m_1]$. For message m_l, the signal is

$$s_l = (-1)^l \sqrt{E}\,; \qquad\qquad l = 0,\, 1\,.$$

The optimum decision rule seeks to determine that l for which the a posteriori (conditional) probability of m_l, given r_1, and r_2, is maximum.

(a) Determine the structure of the optimum decision device and calculate the resulting probability of error.

(b) Compare this result for $\sigma_1 = \sigma_2$ and $P[m_0] = P[m_1]$ with the performance obtained with an optimum decision based only on r_1.

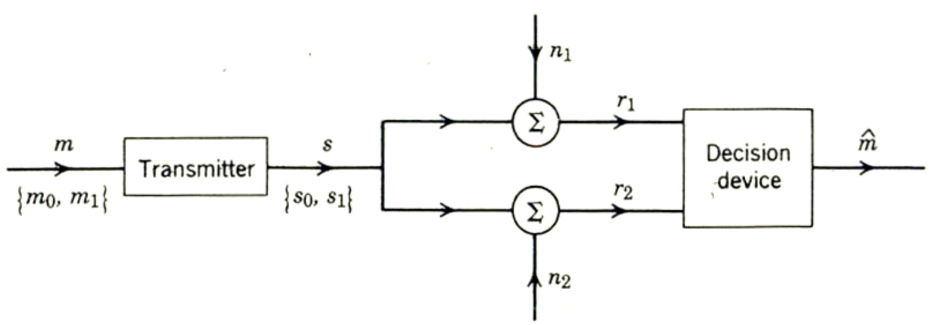

Figure 1:

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3. We consider a probability system on which a random variable x and two arbitrary disjoint events A and B (i.e. $A \cap B = \emptyset$) are defined. The mixed form joint density functions of the random variable x and events A, B are:

$$p_x(\alpha,\, A) = \frac{1}{4}\mathrm{e}^{-\alpha}u(\alpha)$$

$$p_x(\alpha,\, B) = \begin{cases} 1/2 & ; \quad 0 < \alpha < 1 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

   (a) Show that $P(A) = 1/4$ and calculate $P(A \cup B)$.

   (b) Calculate $P(\{x \geq 1/2\} \,|\, A)$.

   (c) Knowing that

$$p_x(1/2) = \frac{11 + 4\mathrm{e}^{-1/2}}{16} \approx 0.839133$$

   (you are not required to show this), calculate $P(A \,|\, x = 1/2)$.

END

Table of the $Q(\quad)$ and $\mathrm{erf}(\quad)$ functions

The approximation $Q(x) \approx \frac{1}{x\sqrt{2\pi}}(1 - \frac{0.7}{x^2})e^{-x^2/2}$ may be used when $x > 2$.

x	erf(x)	$Q(x)$	x	erf(x)	$Q(x)$	x	erf(x)	$Q(x)$
0.00	0	0.5	1.70	0.9838	0.04457	3.40	1	0.0003369
0.10	0.1125	0.4602	1.80	0.9891	0.03593	3.50	1	0.0002326
0.20	0.2227	0.4207	1.90	0.9928	0.02872	3.60	1	0.0001591
0.30	0.3286	0.3821	2.00	0.9953	0.02275	3.70	1	0.0001078
0.40	0.4284	0.3446	2.10	0.997	0.01786	3.80	1	7.235×10^{-5}
0.50	0.5205	0.3085	2.20	0.9981	0.0139	3.90	1	4.810×10^{-5}
0.60	0.6039	0.2743	2.30	0.9989	0.01072	4.00	1	3.167×10^{-5}
0.70	0.6778	0.242	2.40	0.9993	0.008198	4.10	1	2.066×10^{-5}
0.80	0.7421	0.2119	2.50	0.9996	0.00621	4.20	1	1.335×10^{-5}
0.90	0.7969	0.1841	2.60	0.9998	0.004661	4.30	1	8.540×10^{-6}
1.00	0.8427	0.1587	2.70	0.9999	0.003467	4.40	1	5.413×10^{-6}
1.10	0.8802	0.1357	2.80	0.9999	0.002555	4.50	1	3.398×10^{-6}
1.20	0.9103	0.1151	2.90	1	0.001866	4.60	1	2.112×10^{-6}
1.30	0.934	0.0968	3.00	1	0.00135	4.70	1	1.301×10^{-6}
1.40	0.9523	0.08076	3.10	1	0.0009676	4.80	1	7.933×10^{-7}
1.50	0.9661	0.06681	3.20	1	0.0006871	4.90	1	4.792×10^{-7}
1.60	0.9763	0.0548	3.30	1	0.0004834	5.00	1	2.867×10^{-7}