

Name: _____

College Number: _____

EE501: An Introduction to the Theory of Statistical Communications

Thursday, 13 October 2011

Second Quiz

- REMARKS:
1. Hand held calculator is allowed,
 2. Open book quiz, but problem solutions are not allowed,
 3. A table of the $Q(\)$ and $\text{erf}(\)$ functions is attached,
 4. Marks distribution:
 - Question #1: 3 points
 - Question #2: 3 points
 - Question #3: 4 points
 5. Justify all your answers.

# 1	
# 2	
# 3	

1. ¹Two statistically independent Gaussian random variables x_1 and x_2 are applied at the inputs to a threshold detector, the output from which is equal to the number of inputs that exceed the threshold $T = 1$. Thus $y = 0, 1$ or 2 .

$$p_{x_1}(\alpha) = \frac{1}{2\sqrt{2\pi}} e^{-(\alpha-2)^2/8}$$
$$p_{x_2}(\alpha) = \frac{1}{4\sqrt{2\pi}} e^{-(\alpha+2)^2/32}$$

Calculate the following:

- (a) $P(\{\omega : x_1(\omega) < 1\})$
- (b) $P(\{\omega : x_2(\omega) < 1\})$
- (c) $P(\{\omega : x_1(\omega) \geq 1 \text{ and } x_2(\omega) \geq 1\})$
- (d) Determine the density function $p_y(\alpha)$.

¹based on problem 2.14, page 119 in the textbook by W&J.

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2. Figure 1 shows a sketch of the function

$$f(\alpha) = e^{-\alpha/b}u(\alpha) + a\delta(\alpha - 1)$$

where the parameters $a, b \geq 0$ and $u(\cdot)$ denotes the unit step function:

$$u(\alpha) = \begin{cases} 1 & ; \alpha \geq 0 \\ 0 & ; \alpha < 0 \end{cases}$$

- (a) For what range of the parameters a, b is the function $f(\alpha)$ a valid probability density function and what condition(s) must they satisfy?
- (b) Calculate $P(1/2 \leq x \leq 2)$ where x is a random variable with probability density function

$$p_x(\alpha) = e^{-2\alpha}u(\alpha) + \frac{1}{2}\delta(\alpha - 1)$$

- (c) Calculate and sketch the distribution function $F_x(\alpha)$ of the random variable x of question 2b.

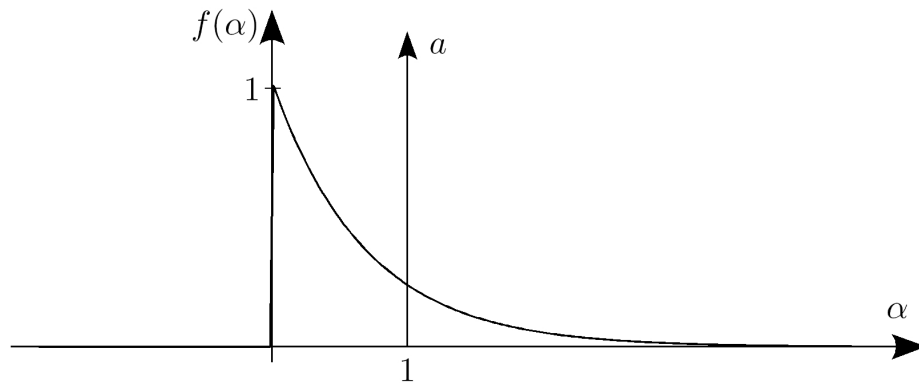


Figure 1:

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3. The joint probability density function of two random variables x and y is given by:

$$p_{xy}(\alpha, \beta) = \begin{cases} \frac{9 - (\alpha - \beta)^2}{36} & ; \quad -3 \leq \alpha - \beta < 3 \\ & \text{and } 0 \leq \beta < 1 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

$p_{xy}(\alpha, \beta)$ is sketched in 3-dimensions in figure 2 and in the colour plot of figure 3. We recall that

$$\int 9 - (\alpha - \beta)^2 d\alpha = 9\alpha - \frac{(\alpha - \beta)^3}{3} + K$$

where $K \in \mathbb{R}$ is a constant.

(a) Show that the random variable y is uniform in the interval $[0, 1]$, i.e. its marginal probability density function $p_y(\beta)$ is given by:

$$p_y(\beta) = \begin{cases} 1 & ; \quad 0 \leq \beta < 1 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

(b) Calculate the probability of the event $\{x \geq 3.5\}$ if it is known that $y = 3/4$.

(c) Calculate the probability density function $p_z(\gamma)$ of the random variable z defined by the transformation $z = 1 - 2y$.

Note: You need not justify your answer, but are encouraged to do so.

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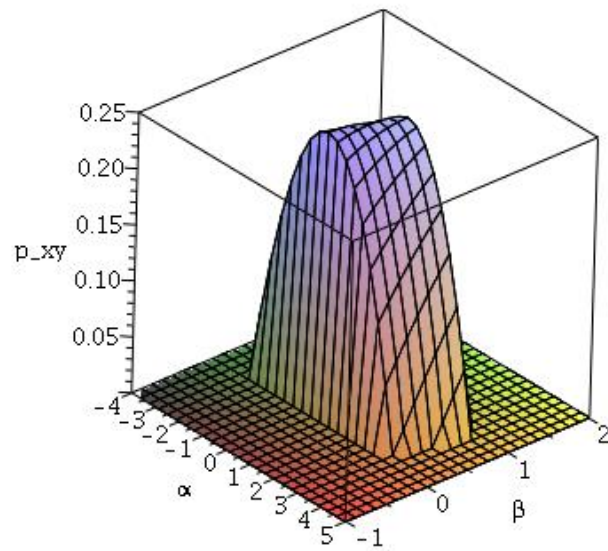


Figure 2:

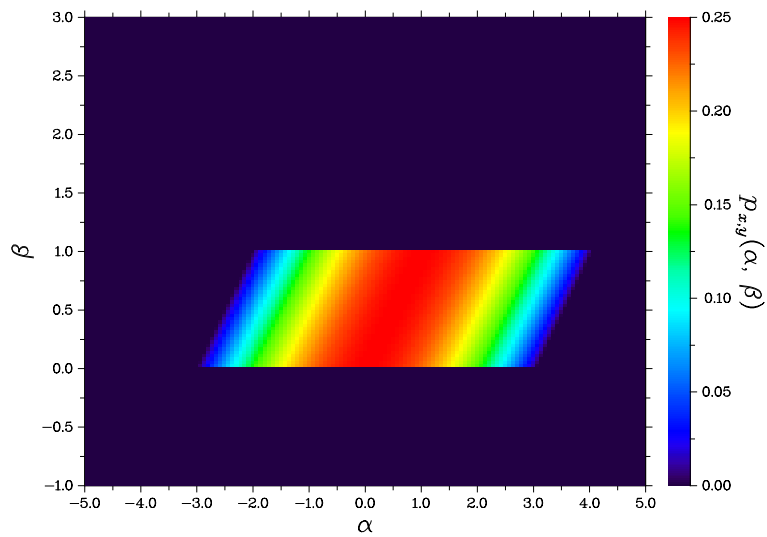


Figure 3:

END

Table of the $Q(x)$ and $\text{erf}(x)$ functions

The approximation $Q(x) \approx \frac{1}{x\sqrt{2\pi}}(1 - \frac{0.7}{x^2})e^{-x^2/2}$ may be used when $x > 2$.

x	$\text{erf}(x)$	$Q(x)$	x	$\text{erf}(x)$	$Q(x)$	x	$\text{erf}(x)$	$Q(x)$
0.00	0	0.5	1.70	0.9838	0.04457	3.40	1	0.0003369
0.10	0.1125	0.4602	1.80	0.9891	0.03593	3.50	1	0.0002326
0.20	0.2227	0.4207	1.90	0.9928	0.02872	3.60	1	0.0001591
0.30	0.3286	0.3821	2.00	0.9953	0.02275	3.70	1	0.0001078
0.40	0.4284	0.3446	2.10	0.997	0.01786	3.80	1	7.235×10^{-5}
0.50	0.5205	0.3085	2.20	0.9981	0.0139	3.90	1	4.810×10^{-5}
0.60	0.6039	0.2743	2.30	0.9989	0.01072	4.00	1	3.167×10^{-5}
0.70	0.6778	0.242	2.40	0.9993	0.008198	4.10	1	2.066×10^{-5}
0.80	0.7421	0.2119	2.50	0.9996	0.00621	4.20	1	1.335×10^{-5}
0.90	0.7969	0.1841	2.60	0.9998	0.004661	4.30	1	8.540×10^{-6}
1.00	0.8427	0.1587	2.70	0.9999	0.003467	4.40	1	5.413×10^{-6}
1.10	0.8802	0.1357	2.80	0.9999	0.002555	4.50	1	3.398×10^{-6}
1.20	0.9103	0.1151	2.90	1	0.001866	4.60	1	2.112×10^{-6}
1.30	0.934	0.0968	3.00	1	0.00135	4.70	1	1.301×10^{-6}
1.40	0.9523	0.08076	3.10	1	0.0009676	4.80	1	7.933×10^{-7}
1.50	0.9661	0.06681	3.20	1	0.0006871	4.90	1	4.792×10^{-7}
1.60	0.9763	0.0548	3.30	1	0.0004834	5.00	1	2.867×10^{-7}