Name: ____________________ College Number: ____________________

EE501: An Introduction to the Theory of Statistical Communications

Thursday, 14 October 2010

Second Quiz

REMARKS:

1. Hand held calculator is allowed,
2. Open book quiz, but problem solutions are not allowed,
3. A table of the $Q(\ )$ and $\mathrm{erf}(\ )$ functions is attached,
4. Marks distribution:
   Question #1: 3 points
   Question #2: 3 points
   Question #3: 4 points
5. Justify all your answers.

# 1	
# 2	
# 3	

1. The probability density function of a Gaussian random variable x is given by:

$$p_x(\alpha) = \frac{1}{5\sqrt{\pi}} e^{-(\alpha-2)^2/25}$$

   Calculate the following:

   (a) $P(\{\omega : x(\omega) < -4\})$
   (b) $P(\{\omega : x(\omega) \geq 4\})$
   (c) $P(\{\omega : -1 \leq x(\omega) < 3\})$

2. We consider a probability system on which a random variable x and two arbitrary disjoint events A and B (i.e. $A \cap B = \emptyset$) are defined. The mixed form joint density functions of the random variable x and events A, B are:

$$p_x(\alpha,\, A) = \frac{1}{4} e^{-\alpha} u(\alpha)$$

$$p_x(\alpha,\, B) = \begin{cases} 1/2 & ; \quad 0 < \alpha < 1 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

   (a) Show that $P(A) = 1/4$ and calculate $P(A \cup B)$.
   (b) Calculate $P(\{x \geq 1/2\} \,|\, A)$.
   (c) Knowing that

$$p_x(1/2) = \frac{11 + 4e^{-1/2}}{16} \approx 0.839133$$

   (you are not required to show this), calculate $P(A \,|\, x = 1/2)$.

3. The joint probability density function of two random variables x and y is given by:

$$p_{xy}(\alpha,\beta) = \begin{cases} \frac{9-(\alpha-\beta)^2}{36} & ; \quad -3 \leq \alpha - \beta < 3 \\ & \quad \text{and } 0 \leq \beta < 1 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

$p_{xy}(\alpha,\ \beta)$ is sketched in 3-dimensions in figure 1 and in the colour plot of figure 2. We recall that

$$\int 9 - (\alpha-\beta)^2 \mathrm{d}\alpha = 9\alpha - \frac{(\alpha-\beta)^3}{3} + K$$

where $K \in \mathbb{R}$ is a constant.

(a) Show that the random variable y is uniform in the interval $[0,\ 1]$, i.e. its marginal probability density function $p_y(\beta)$ is given by:

$$p_y(\beta) = \begin{cases} 1 & ; \quad 0 \leq \beta < 1 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

(b) Calculate the probability of the event $\{x \geq 3.5\}$ if it is known that $y = 3/4$.

(c) Calculate the probability density function $p_z(\gamma)$ of the random variable z defined by the transformation $z = 1 - 2y$.
Note: You need not justify your answer, but are encouraged to do so.

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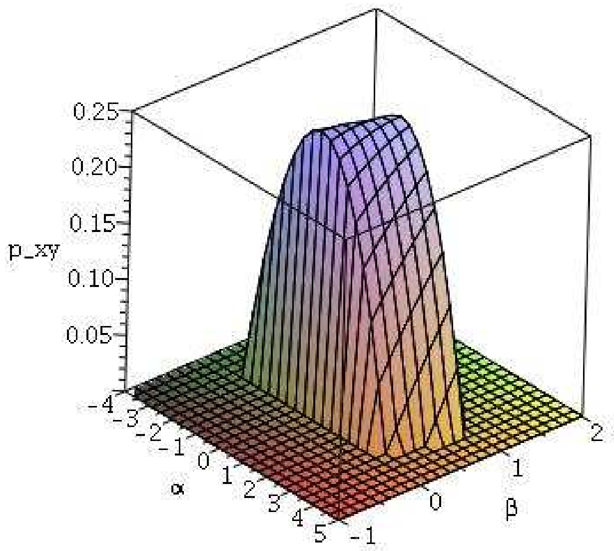

Figure 1:

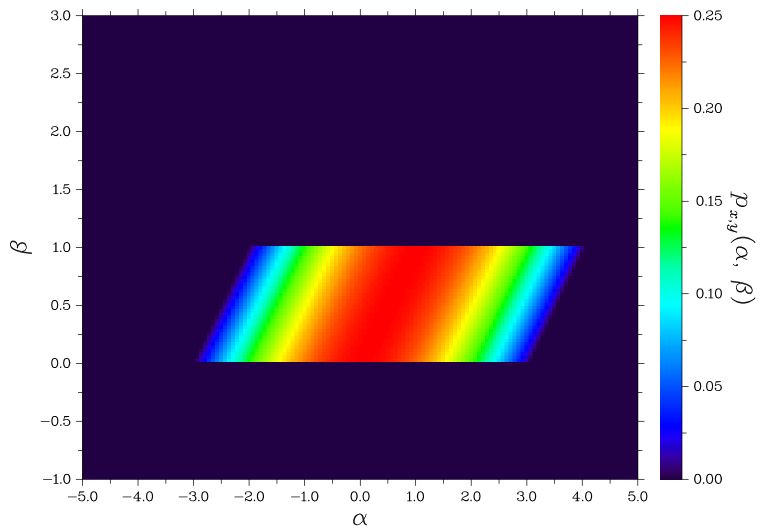

Figure 2:

END

Table of the $Q(\quad)$ and $\mathrm{erf}(\quad)$ functions

The approximation $Q(x) \approx \frac{1}{x\sqrt{2\pi}}(1 - \frac{0.7}{x^2})e^{-x^2/2}$ may be used when $x > 2$.

x	$\mathrm{erf}(x)$	$Q(x)$	x	$\mathrm{erf}(x)$	$Q(x)$	x	$\mathrm{erf}(x)$	$Q(x)$
0.00	0	0.5	1.70	0.9838	0.04457	3.40	1	0.0003369
0.10	0.1125	0.4602	1.80	0.9891	0.03593	3.50	1	0.0002326
0.20	0.2227	0.4207	1.90	0.9928	0.02872	3.60	1	0.0001591
0.30	0.3286	0.3821	2.00	0.9953	0.02275	3.70	1	0.0001078
0.40	0.4284	0.3446	2.10	0.997	0.01786	3.80	1	7.235×10^{-5}
0.50	0.5205	0.3085	2.20	0.9981	0.0139	3.90	1	4.810×10^{-5}
0.60	0.6039	0.2743	2.30	0.9989	0.01072	4.00	1	3.167×10^{-5}
0.70	0.6778	0.242	2.40	0.9993	0.008198	4.10	1	2.066×10^{-5}
0.80	0.7421	0.2119	2.50	0.9996	0.00621	4.20	1	1.335×10^{-5}
0.90	0.7969	0.1841	2.60	0.9998	0.004661	4.30	1	8.540×10^{-6}
1.00	0.8427	0.1587	2.70	0.9999	0.003467	4.40	1	5.413×10^{-6}
1.10	0.8802	0.1357	2.80	0.9999	0.002555	4.50	1	3.398×10^{-6}
1.20	0.9103	0.1151	2.90	1	0.001866	4.60	1	2.112×10^{-6}
1.30	0.934	0.0968	3.00	1	0.00135	4.70	1	1.301×10^{-6}
1.40	0.9523	0.08076	3.10	1	0.0009676	4.80	1	7.933×10^{-7}
1.50	0.9661	0.06681	3.20	1	0.0006871	4.90	1	4.792×10^{-7}
1.60	0.9763	0.0548	3.30	1	0.0004834	5.00	1	2.867×10^{-7}