Name: ______________________ College Number: ______________________

EE501: An Introduction to the Theory of Statistical Communications

Tuesday, 25 September 2018

First Quiz

REMARKS:
1. Hand held calculator is allowed,
2. Open book quiz,
3. A table of the $Q(\ )$ and $\mathrm{erf}(\ )$ functions is attached,
4. Marks distribution:
   Question #1: 3 points
   Question #2: 4 points
   Question #3: 3 points
5. Justify all your answers.

# 1	
# 2	
# 3	

1. Consider a probability system $(\Omega, \mathscr{F}, P : \mathscr{F} \to [0,1])$ and a Gaussian random variable $x : \Omega \to \mathbb{R} \cup \{\pm\infty\}$ with probability density function:

$$p_x(\alpha) = \frac{1}{4\sqrt{2\pi}} \mathrm{e}^{-(\alpha-2)^2/32}$$

Calculate $P\big(A \cup \big((\overline{B} \cap D) \cap \overline{C}\big) \cup E\big)$ where the events A, B, C, D, E are given by:

$$\begin{aligned} A &= \{\omega : x(\omega) < -6\} \\ B &= \{\omega : x(\omega) < -4\} \\ C &= \{\omega : x(\omega) \geq -1\} \\ D &= \{\omega : x(\omega) < 3\} \\ E &= \{\omega : x(\omega) \geq 4\} \end{aligned}$$

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2. A symbol from the three-symbol alphabet $\{m_0, m_1, m_2\}$ is transmitted over a noisy discrete communication channel and a symbol from the two-symbol alphabet $\{r_0, r_1\}$ is received. The transition probabilities of the channel are specified by the diagram of figure 1. A source is available that uses m_0, m_1, and m_2 with the following probabilities:

$$\begin{aligned} P(m_0) &= 0.4, \\ P(m_1) &= 0.3, \\ P(m_2) &= 0.3. \end{aligned}$$

   (a) What is the optimum receiver decision rule (assignment of r_0, r_1 to m_0, m_1, m_2)?

   (b) For the decision rule $\widehat{m}_G$ given by (not necessarily optimal):

$$\begin{array}{llcl} \widehat{m}_G : & \{r_0, r_1\} & \to & \{m_0, m_1, m_2\} \\ \widehat{m}_G : & r_0 & \mapsto & m_2 \\ \widehat{m}_G : & r_1 & \mapsto & m_0 \end{array}$$

   calculate the probability that the receiver's decision is m_2.

   (c) For the decision rule $\widehat{m}_G$ used in subquestion (2b) above, calculate the probability of error.

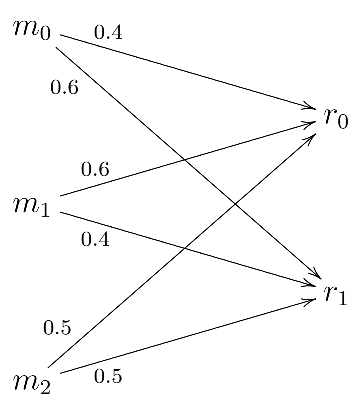

Figure 1:

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3. Calculate and sketch the probability density function $p_x(\alpha)$ of the random variable x with the probability distribution function sketched in figure 2.

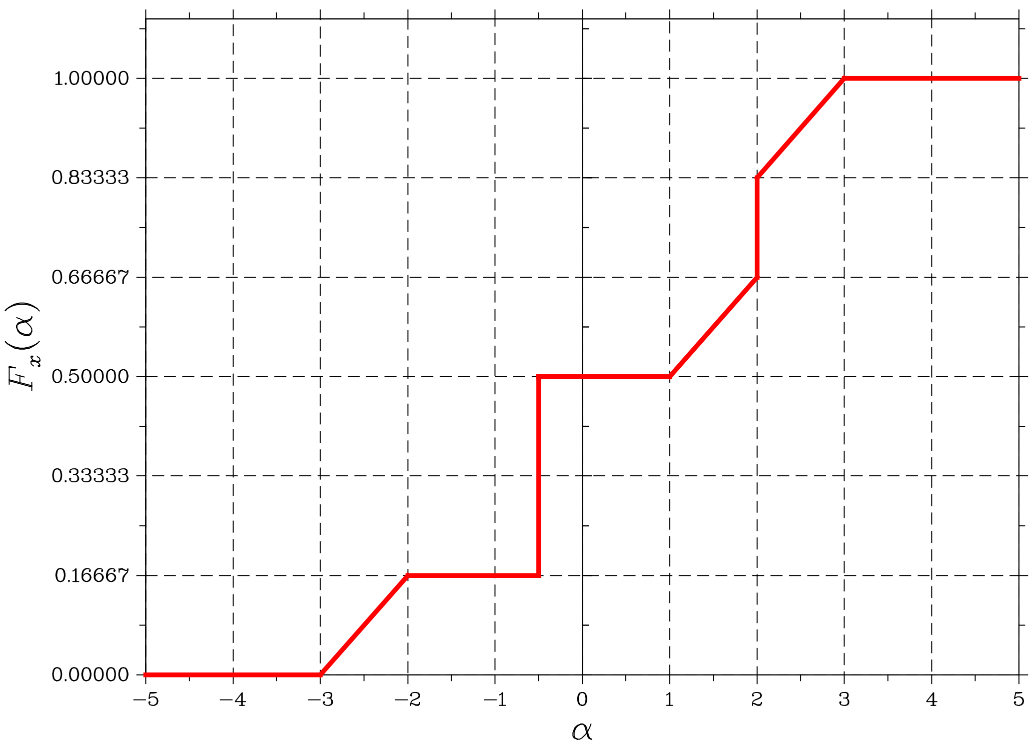

Figure 2:

END

Table of the $Q(\quad)$ and $\mathrm{erf}(\quad)$ functions

The approximation $Q(x) \approx \frac{1}{x\sqrt{2\pi}}(1 - \frac{0.7}{x^2})e^{-x^2/2}$ may be used when $x > 2$.

x	erf(x)	$Q(x)$	x	erf(x)	$Q(x)$	x	erf(x)	$Q(x)$
0.00	0	0.5	1.70	0.9838	0.04457	3.40	1	0.0003369
0.10	0.1125	0.4602	1.80	0.9891	0.03593	3.50	1	0.0002326
0.20	0.2227	0.4207	1.90	0.9928	0.02872	3.60	1	0.0001591
0.30	0.3286	0.3821	2.00	0.9953	0.02275	3.70	1	0.0001078
0.40	0.4284	0.3446	2.10	0.997	0.01786	3.80	1	7.235×10^{-5}
0.50	0.5205	0.3085	2.20	0.9981	0.0139	3.90	1	4.810×10^{-5}
0.60	0.6039	0.2743	2.30	0.9989	0.01072	4.00	1	3.167×10^{-5}
0.70	0.6778	0.242	2.40	0.9993	0.008198	4.10	1	2.066×10^{-5}
0.80	0.7421	0.2119	2.50	0.9996	0.00621	4.20	1	1.335×10^{-5}
0.90	0.7969	0.1841	2.60	0.9998	0.004661	4.30	1	8.540×10^{-6}
1.00	0.8427	0.1587	2.70	0.9999	0.003467	4.40	1	5.413×10^{-6}
1.10	0.8802	0.1357	2.80	0.9999	0.002555	4.50	1	3.398×10^{-6}
1.20	0.9103	0.1151	2.90	1	0.001866	4.60	1	2.112×10^{-6}
1.30	0.934	0.0968	3.00	1	0.00135	4.70	1	1.301×10^{-6}
1.40	0.9523	0.08076	3.10	1	0.0009676	4.80	1	7.933×10^{-7}
1.50	0.9661	0.06681	3.20	1	0.0006871	4.90	1	4.792×10^{-7}
1.60	0.9763	0.0548	3.30	1	0.0004834	5.00	1	2.867×10^{-7}