Name: _____ College Number: _____

EE501: An Introduction to the Theory of Statistical Communications

Tuesday, 27 September 2011

First Quiz

REMARKS:
1. Hand held calculator is allowed,
2. Open book quiz (no solutions to the problems),
3. Marks distribution:
   Question #1: 3 points
   Question #2: 4 points
   Question #3: 3 points
4. Justify all your answers.

# 1	
# 2	
# 3	

1. A probability system $(\Omega, \mathscr{F}, P(\,))$ is defined by:
   - $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8\}$,
   - $\mathscr{F} = 2^{\Omega}$,
   - $P(\,)$ is defined by:
   $$\begin{array}{rccc} P: & \mathscr{F} & \rightarrow & [0,\,1] \\ P: & E & \mapsto & \frac{|E|}{8} \end{array}$$
   or in other words the events consisting of a single sample point are equiprobable.

   Consider the following events A, B, $C \in \mathscr{F}$:
   $$\begin{aligned} A &= \{\,1,\,2,\,3,\,4\,\} \\ B &= \{\,2,\,3,\,5,\,6\,\} \\ C &= \{\,3,\,4,\,6,\,7\,\} \end{aligned}$$
   (a) Are A, B, C statistically independent?
   (b) Are the events $E = \overline{A} \cup B$ and $F = \overline{C}$ statistically independent?

2. A noisy discrete communication channel is available. A symbol from the three-symbol alphabet $\{m_0, m_1, m_2\}$ is transmitted and a symbol from the two-symbol alphabet $\{r_1, r_2\}$, received. The conditional probabilities of the various received symbols, given the various transmitted symbols, are specified by the diagram of figure 1. A source is available that uses m_0, m_1, and m_2 with the following probabilities:
   $$\begin{aligned} P(m_0) &= 1/3, \\ P(m_1) &= 1/3, \\ P(m_2) &= 1/3. \end{aligned}$$

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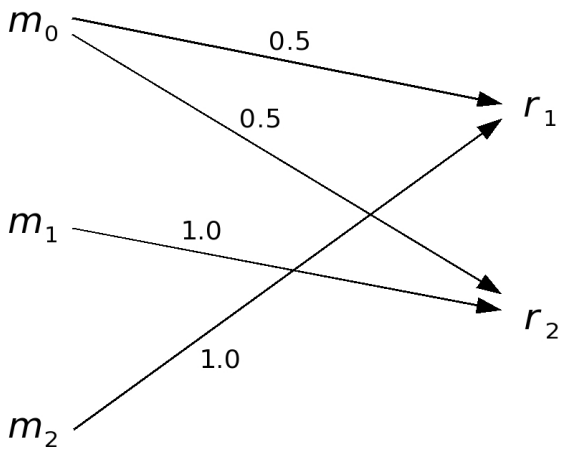

Figure 1:

What is the best receiver decision rule (assignment of r_1, r_2 to m_0, m_1, m_2) and what is the resulting probability of error?

3. Consider the probability space defined by:[1]

   - the sample space is the real-line interval $\Omega = \{\, \omega \,:\, 0 \leq \omega \leq 1 \,\}$;
   - the class of events is the set of all intervals of this line segment, plus (countable) unions, (countable) intersections, and complements of such intervals. The intervals may include both, one, or either of the end points;
   - the probability assignment is the sum of the lengths of the disjoint intervals that constitute the event.

   Note: You need not show that x is a *valid* random variable.

   (a) Calculate the probability distribution function $F_x(\alpha)$ of the random variable x defined by:

   $$\begin{array}{rccc} x: & \Omega & \rightarrow & \mathbb{R} \\ x: & \omega & \mapsto & \frac{1}{1-\omega} \end{array}$$

   (b) Using $F_x(\alpha)$, calculate $P(\{\, \omega \,:\, 2 < x(\omega) \leq 3 \,\})$.

END

[1] W&J, bottom of page 21 and top of page 22.